Opti 501 **Solutions**

Problem 28)

This problem is similar to problem 27, except that in the substrate $n_s \neq 1$. Then $\sigma_z'' = -n_s$ and $Z_0 \vec{H}_0'' = -n_s \tau E_x \hat{y}$. The matching of E_x and H_y fields at the top interface yields the same expressions for A and B (in terms of n_0 and r) as before. However, at the bottom interface the equations are somewhat different. We'll find:

$$\begin{cases} \frac{1}{2n_0}\left[(n_0+1)+(n_0-1)r\right]e^{2ik_0 n_0 d} + \frac{1}{2n_0}\left[(n_0-1)+(n_0+1)r\right] = e^{ik_0(n_0+n_s)d}\tau \\ \frac{1}{2n_s}\left[(n_0+1)+(n_0-1)r\right]e^{2ik_0 n_0 d} - \frac{1}{2n_s}\left[(n_0-1)+(n_0+1)r\right] = e^{ik_0(n_0+n_s)d}\tau \end{cases}$$

Again, we set the left-hand-sides of the above equations equal to each other:

$$\left(\frac{1}{n_0}-\frac{1}{n_s}\right)\left[(n_0+1)+(n_0-1)r\right]e^{2ik_0 n_0 d} + \left(\frac{1}{n_0}+\frac{1}{n_s}\right)\left[(n_0-1)+(n_0+1)r\right] = 0 \Rightarrow$$

$$(n_s-n_0)\cancel{(n_0+1)}\left[1+\left(\frac{n_0-1}{n_0+1}\right)r\right]e^{2ik_0 n_0 d} + (n_0+n_s)\cancel{(n_0+1)}\left[\left(\frac{n_0-1}{n_0+1}\right)+r\right] = 0 \Rightarrow$$

$$-\left(\frac{n_0-n_s}{n_0+n_s}\right)\left[1-\left(\frac{1-n_0}{1+n_0}\right)r\right]e^{2ik_0 n_0 d} - \left(\frac{1-n_0}{1+n_0}\right) + r = 0 \Rightarrow$$

$$r = \frac{\left(\frac{1-n_0}{1+n_0}\right)+\left(\frac{n_0-n_s}{n_0+n_s}\right)e^{i4\pi n_0 d/\lambda_0}}{1+\left(\frac{1-n_0}{1+n_0}\right)\left(\frac{n_0-n_s}{n_0+n_s}\right)e^{i4\pi n_0 d/\lambda_0}}$$

→ To determine A, B, and τ substitute for r in the preceding equations.

Next let $n_0 = \sqrt{n_s}$ and $d = \lambda_0/4n_0$. We'll have:

$$r = \frac{\left(\frac{1-\sqrt{n_s}}{1+\sqrt{n_s}}\right)-\left(\frac{\sqrt{n_s}-n_s}{\sqrt{n_s}+n_s}\right)}{1-\left(\frac{1-\sqrt{n_s}}{1+\sqrt{n_s}}\right)\left(\frac{\sqrt{n_s}-n_s}{\sqrt{n_s}+n_s}\right)} = 0 \leftarrow \text{Antireflection Coating} \checkmark$$