Opti 501 **Solutions**

Problem 16)

$$k_x = (\omega/c)\sqrt{\mu_a\epsilon_a}\sin\theta \Rightarrow (ck_x/\omega)^2 = \mu_a\epsilon_a\sin^2\theta$$

$$\rho_p = \frac{\epsilon_a\sqrt{\mu_b\epsilon_b-(ck_x/\omega)^2} - \epsilon_b\sqrt{\mu_a\epsilon_a-(ck_x/\omega)^2}}{\epsilon_a\sqrt{\mu_b\epsilon_b-(ck_x/\omega)^2} + \epsilon_b\sqrt{\mu_a\epsilon_a-(ck_x/\omega)^2}}$$

Plane-electromagnetic waves ← Eq. (17a)

$$\rho_s = \frac{\mu_b\sqrt{\mu_a\epsilon_a-(ck_x/\omega)^2} - \mu_a\sqrt{\mu_b\epsilon_b-(ck_x/\omega)^2}}{\mu_b\sqrt{\mu_a\epsilon_a-(ck_x/\omega)^2} + \mu_a\sqrt{\mu_b\epsilon_b-(ck_x/\omega)^2}}$$ ← Eq. (19a)

a) $n_a = n_b \Rightarrow \mu_a\epsilon_a = \mu_b\epsilon_b \Rightarrow \frac{\epsilon_a}{\epsilon_b} = \frac{\mu_b}{\mu_a}$

$$\Rightarrow \rho_p = \frac{\epsilon_a-\epsilon_b}{\epsilon_a+\epsilon_b} \quad \text{and} \quad \rho_s = \frac{\mu_b-\mu_a}{\mu_b+\mu_a} \quad \checkmark$$

However, $\rho_p = \dfrac{\epsilon_a/\epsilon_b - 1}{\epsilon_a/\epsilon_b + 1} = \dfrac{\frac{\mu_b}{\mu_a}-1}{\frac{\mu_b}{\mu_a}+1} = \dfrac{\mu_b-\mu_a}{\mu_b+\mu_a} = \rho_s$ ✓

Thus $\rho_p = \rho_s$ not only at normal incidence, but also at all angles of incidence $0 \le \theta \le 90°$. In fact, ρ_p and ρ_s are independent of θ and have the same values irrespective of the angle of incidence.

b) $\frac{\mu_a}{\epsilon_a} = \frac{\mu_b}{\epsilon_b} \Rightarrow \mu_a\epsilon_b = \mu_b\epsilon_a$

$$\Rightarrow \rho_p = \frac{\epsilon_a\sqrt{\mu_b\epsilon_b-\mu_a\epsilon_a\sin^2\theta} - \epsilon_b\sqrt{\mu_a\epsilon_a}\cos\theta}{\epsilon_a\sqrt{\mu_b\epsilon_b-\mu_a\epsilon_a\sin^2\theta} + \epsilon_b\sqrt{\mu_a\epsilon_a}\cos\theta} = 0 \Rightarrow$$

$$\epsilon_a\sqrt{\mu_b\epsilon_b-\mu_a\epsilon_a\sin^2\theta} = \epsilon_b\sqrt{\mu_a\epsilon_a}\cos\theta \Rightarrow \epsilon_a^2(\mu_b\epsilon_b-\mu_a\epsilon_a\sin^2\theta) = \epsilon_b^2\mu_a\epsilon_a\cos^2\theta$$

$$\Rightarrow \epsilon_a\mu_b\epsilon_b - \mu_a\epsilon_a^2\sin^2\theta = \mu_a\epsilon_b^2(1-\sin^2\theta) \Rightarrow \epsilon_b(\epsilon_a\mu_b-\mu_a\epsilon_b) = \mu_a(\epsilon_a^2-\epsilon_b^2)\sin^2\theta$$

Equality of impedances means that $\epsilon_a \mu_b = \mu_a \epsilon_b$ and, therefore, the left-hand side of the above equation is zero. Consequently the right-hand side must be zero, which means that $\sin\theta = 0$. We conclude that only at normal incidence, when $\theta = 0$, it is possible to have $\rho_p = 0$.

$$\rho_s = 0 \Rightarrow \mu_b \sqrt{\mu_a \epsilon_a - \mu_a \epsilon_a \sin^2\theta} - \mu_a \sqrt{\mu_b \epsilon_b - \mu_a \epsilon_a \sin^2\theta} = 0 \Rightarrow$$

$$\mu_b^2 \mu_a \epsilon_a \cos^2\theta = \mu_a^2 (\mu_b \epsilon_b - \mu_a \epsilon_a \sin^2\theta) \Rightarrow \mu_b^2 \epsilon_a (1 - \sin^2\theta) = \mu_a \mu_b \epsilon_b - \mu_a^2 \epsilon_a \sin^2\theta$$

$$\Rightarrow \mu_b \cancelto{0}{(\mu_b \epsilon_a - \mu_a \epsilon_b)} = \epsilon_a (\mu_b^2 - \mu_a^2) \sin^2\theta \Rightarrow \sin\theta = 0 \leftarrow \text{only at normal incidence is } \rho_s = 0$$