Problem 13) a) $\boldsymbol{H}(x,t) = H_0 \cos\{\omega[t - n(\omega)x/c]\}\,\hat{\boldsymbol{z}}$.

Maxwell's third equation: $\boldsymbol{\nabla} \times \boldsymbol{E} = -\,\partial \boldsymbol{B}/\partial t \quad \rightarrow \quad (\partial E_y/\partial x)\hat{\boldsymbol{z}} = -\mu_0(\partial H_z/\partial t)\hat{\boldsymbol{z}}$

$$\rightarrow \quad E_0[\omega n(\omega)/c]\sin\{\omega[t - n(\omega)x/c]\} = \mu_0 H_0 \omega \sin\{\omega[t - n(\omega)x/c]\}$$

$$\rightarrow \quad H_0 = n(\omega)E_0/\mu_0 c = n(\omega)E_0/Z_0.$$

b)
$$\boldsymbol{S}(x,t) = \boldsymbol{E}(x,t) \times \boldsymbol{H}(x,t) = E_0 H_0 \cos^2\{\omega[t - n(\omega)x/c]\}\,\hat{\boldsymbol{x}}$$

$$\rightarrow \quad \langle \boldsymbol{S}(x,t) \rangle = E_0 H_0 \langle \cos^2\{\omega[t - n(\omega)x/c]\}\rangle \hat{\boldsymbol{x}} = [n(\omega)/2Z_0]E_0^2 \hat{\boldsymbol{x}}.$$

c)
$$\boldsymbol{E}_1(x,t) + \boldsymbol{E}_2(x,t) = E_0\{\cos\{\omega[t - n(\omega)x/c]\} + \cos\{\omega'[t - n(\omega')x/c]\}\}\hat{\boldsymbol{y}}$$

$$= 2E_0 \cos\left\{\left(\frac{\omega + \omega'}{2}\right)t - \frac{[\omega n(\omega) + \omega' n(\omega')]x}{2c}\right\} \cos\left\{\left(\frac{\omega' - \omega}{2}\right)t - \frac{[\omega' n(\omega') - \omega n(\omega)]x}{2c}\right\}\hat{\boldsymbol{y}}$$

$$\cong 2E_0 \underbrace{\cos\{\omega_c[t - n(\omega_c)x/c]\}}_{\substack{\text{carrier:} \\ \text{phase velocity} = c/n(\omega_c)}} \underbrace{\cos\left\{\tfrac{1}{2}\Delta\omega\left[t - \frac{\omega' n(\omega') - \omega n(\omega)}{\omega' - \omega}(x/c)\right]\right\}}_{\substack{\text{envelope:} \\ \text{group velocity} = \frac{c}{\mathrm{d}[\omega n(\omega)]/\mathrm{d}\omega|_{\omega=\omega_c}}}}\hat{\boldsymbol{y}}.$$

Here, $\mathrm{d}[\omega n(\omega)]/\mathrm{d}\omega|_{\omega=\omega_c} = n(\omega_c) + \omega_c n'(\omega_c)$, where the derivative n' of the refractive index n is evaluated at the center frequency ω_c.