

**Problem 8)**

a)  $\rho_p=0 \rightarrow \varepsilon_a \sqrt{\mu_b \varepsilon_b - (ck_x/\omega)^2} = \varepsilon_b \sqrt{\mu_a \varepsilon_a - (ck_x/\omega)^2}$  where  $k_x = (\omega/c) \sqrt{\mu_a \varepsilon_a} \sin \theta_{Bp}$ . Therefore,

$$\varepsilon_a^2 (\mu_b \varepsilon_b - \mu_a \varepsilon_a \sin^2 \theta_{Bp}) = \varepsilon_b^2 (\mu_a \varepsilon_a - \mu_a \varepsilon_a \sin^2 \theta_{Bp}) \rightarrow \sin^2 \theta_{Bp} = (\varepsilon_b / \mu_a) (\varepsilon_a \mu_b - \varepsilon_b \mu_a) / (\varepsilon_a^2 - \varepsilon_b^2)$$

$$\rightarrow \cos^2 \theta_{Bp} = 1 - \sin^2 \theta_{Bp} = (\varepsilon_a / \mu_a) (\varepsilon_a \mu_a - \varepsilon_b \mu_b) / (\varepsilon_a^2 - \varepsilon_b^2)$$

$$\rightarrow \tan^2 \theta_{Bp} = (\varepsilon_b / \varepsilon_a) (\varepsilon_a \mu_b - \varepsilon_b \mu_a) / (\varepsilon_a \mu_a - \varepsilon_b \mu_b).$$

b)  $\rho_s=0 \rightarrow \mu_a \sqrt{\mu_b \varepsilon_b - (ck_x/\omega)^2} = \mu_b \sqrt{\mu_a \varepsilon_a - (ck_x/\omega)^2}$  where  $k_x = (\omega/c) \sqrt{\mu_a \varepsilon_a} \sin \theta_{Bs}$ . Therefore,

$$\mu_a^2 (\mu_b \varepsilon_b - \mu_a \varepsilon_a \sin^2 \theta_{Bs}) = \mu_b^2 (\mu_a \varepsilon_a - \mu_a \varepsilon_a \sin^2 \theta_{Bs}) \rightarrow \sin^2 \theta_{Bs} = (\mu_b / \varepsilon_a) (\mu_a \varepsilon_b - \mu_b \varepsilon_a) / (\mu_a^2 - \mu_b^2)$$

$$\rightarrow \cos^2 \theta_{Bs} = 1 - \sin^2 \theta_{Bs} = (\mu_a / \varepsilon_a) (\varepsilon_a \mu_a - \varepsilon_b \mu_b) / (\mu_a^2 - \mu_b^2)$$

$$\rightarrow \tan^2 \theta_{Bs} = -(\mu_b / \mu_a) (\varepsilon_a \mu_b - \varepsilon_b \mu_a) / (\varepsilon_a \mu_a - \varepsilon_b \mu_b).$$

c) In the above expressions for  $\tan^2 \theta_{Bp}$  and  $\tan^2 \theta_{Bs}$ , the second and third terms are identical. As for the first terms, the signs of  $\varepsilon_a$  and  $\mu_a$  are generally the same, and so are the signs of  $\varepsilon_b$  and  $\mu_b$ . Therefore, the signs of  $\tan^2 \theta_{Bp}$  and  $\tan^2 \theta_{Bs}$  are going to be opposite, that is, if one is positive, the other will be negative. Since tangent-squared needs to be positive, it will be *impossible* to have Brewster's angles for both p- and s-light.