Problem 6)

a) $\rho_{bound}(\vec{r},t) = -\vec{\nabla}\cdot\vec{P}(\vec{r},t) = -\frac{\partial}{\partial z}P_z(\vec{r},t) = 0$ ✓

$$\vec{J}_{bound}(\vec{r},t) = \frac{\partial}{\partial t}\vec{P}(\vec{r},t) = (P_o\omega\Delta)\cos(\omega t-\phi_o)\hat{z}.$$

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b) This is similar to problem 21, with current density within the xz-plane having magnitude $P_o\omega\Delta$, and a constant phase ϕ_o. We can write the E- and H-fields directly from the solution to problem 21 as follows:

$$\begin{cases} \vec{E}(\vec{r},t) = -\frac{1}{2}(Z_oP_o\omega\Delta)\cos[\omega(t-\frac{|y|}{c})-\phi_o]\hat{z} \\ \vec{H}(\vec{r},t) = -\frac{1}{2}(P_o\omega\Delta)\,\mathrm{Sign}(y)\cos[\omega(t-\frac{|y|}{c})-\phi_o]\hat{x}. \end{cases}$$

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c) The E-field radiated by the dipole-sheet, when evaluated at $y=0$, yields the field that opposes the oscillations of these dipoles. We have:

$$\vec{E}(x,y=0,z,t) = -\frac{1}{2}(Z_oP_o\omega\Delta)\cos(\omega t-\phi_o)\hat{z}.$$

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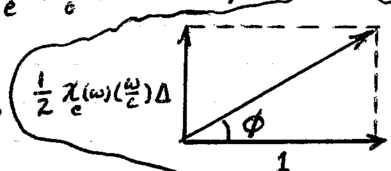
d) Total E-field that drives the dipoles $= [E_o\sin\omega t - \frac{1}{2}(Z_oP_o\omega\Delta)\cos(\omega t-\phi_o)]\hat{z}$.
Considering that the material medium of the sheet has a real-valued susceptibility $\epsilon_o\chi_e(\omega)$, we can write the following relation between the E-field that drives the dipoles and the polarization density $\vec{P}(\vec{r},t)$:

$$P_o\sin(\omega t-\phi_o)\hat{z} = \epsilon_o\chi_e(\omega)[E_o\sin\omega t - \frac{1}{2}(Z_oP_o\omega\Delta)\cos(\omega t-\phi_o)]\hat{z} \Rightarrow$$

$$P_o[\sin(\omega t-\phi_o) + \frac{1}{2}\chi_e(\omega)(\omega/c)\Delta\cos(\omega t-\phi_o)] = \epsilon_o\chi_e(\omega)E_o\sin\omega t \Rightarrow$$

$$P_o\sqrt{1+[\frac{1}{2}\chi_e(\omega)(\omega/c)\Delta]^2}\,[\cos\phi\sin(\omega t-\phi_o)+\sin\phi\cos(\omega t-\phi_o)] = \epsilon_o\chi_e(\omega)E_o\sin\omega t$$



$$\Rightarrow P_o\sqrt{1+[\tfrac{1}{2}\chi_e(\omega)(\omega/c)\Delta]^2}\,\sin(\omega t-\phi_o+\phi)=\epsilon_o\chi_e(\omega)E_o\sin\omega t$$

$$\Rightarrow P_o=\frac{\epsilon_o\chi_e(\omega)E_o}{\sqrt{1+[\tfrac{1}{2}\chi_e(\omega)(\omega/c)\Delta]^2}}\,;\qquad \phi_o=\phi=\sin^{-1}\frac{\tfrac{1}{2}\chi_e(\omega)(\frac{\omega}{c})\Delta}{\sqrt{1+[\tfrac{1}{2}\chi_e(\omega)(\omega/c)\Delta]^2}}$$

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e) Incident $\langle\vec{S}(\vec{r},t)\rangle=\frac{1}{2}\frac{E_o^2}{Z_o}\hat{y}$ ✓

Reflected $\langle\vec{S}(\vec{r},t)\rangle=\frac{1}{2Z_o}\left(\frac{1}{2}Z_oP_o\omega\Delta\right)^2(-\hat{y})=\frac{1}{2Z_o}\frac{[\frac{1}{2}\chi_e(\omega)(\frac{\omega}{c})\Delta]^2E_o^2}{1+[\frac{1}{2}\chi_e(\omega)(\omega/c)\Delta]^2}(-\hat{y})$ ✓

Transmitted E-field amplitude $=E_o\sin[\omega(t-\frac{y}{c})]-\frac{1}{2}(Z_oP_o\omega\Delta)\cos[\omega(t-\frac{y}{c})-\phi_o]$

$$=\left[E_o-\tfrac{1}{2}(Z_oP_o\omega\Delta)\sin\phi_o\right]\sin[\omega(t-\tfrac{y}{c})]-\tfrac{1}{2}(Z_oP_o\omega\Delta)\cos\phi_o\cos[\omega(t-\tfrac{y}{c})]$$

$$=E_o\left\{1-\frac{[\frac{1}{2}\chi_e(\omega)(\omega/c)\Delta]^2}{1+[\frac{1}{2}\chi_e(\omega)(\omega/c)\Delta]^2}\right\}\sin[\omega(t-\tfrac{y}{c})]-\frac{[\frac{1}{2}\chi_e(\omega)(\omega/c)\Delta]E_o}{1+[\frac{1}{2}\chi_e(\omega)(\omega/c)\Delta]^2}\cos[\omega(t-\tfrac{y}{c})]$$

$$=\frac{E_o}{1+[\frac{1}{2}\chi_e(\omega)(\omega/c)\Delta]^2}\left\{\sin[\omega(t-\tfrac{y}{c})]-[\tfrac{1}{2}\chi_e(\omega)(\omega/c)\Delta]\cos[\omega(t-\tfrac{y}{c})]\right\}$$

$$=\frac{E_o}{\sqrt{1+[\frac{1}{2}\chi_e(\omega)(\omega/c)\Delta]^2}}\sin[\omega(t-\tfrac{y}{c})-\phi_o].$$

Therefore, transmitted $\langle\vec{S}(\vec{r},t)\rangle=\frac{1}{2Z_o}\frac{E_o^2}{1+[\frac{1}{2}\chi_e(\omega)(\omega/c)\Delta]^2}\hat{y}.$

Note that the reflected and transmitted optical powers add up to exactly the incident optical power. Conservation of energy is therefore confirmed.