Problem 2)

$$\vec{S}(\vec{r},t) = \vec{E}(\vec{r},t) \times \vec{H}(\vec{r},t) = \{\vec{E}'(\vec{r})\cos\omega_0 t + \vec{E}''(\vec{r})\sin\omega_0 t\} \times$$

$$\{\vec{H}'(\vec{r})\cos\omega_0 t + \vec{H}''(\vec{r})\sin\omega_0 t\} = [\vec{E}'(\vec{r})\times\vec{H}'(\vec{r})]\cos^2\omega_0 t + [\vec{E}''(\vec{r})\times\vec{H}''(\vec{r})]\sin^2\omega_0 t$$

$$+ [\vec{E}'(\vec{r})\times\vec{H}''(\vec{r}) + \vec{E}''(\vec{r})\times\vec{H}'(\vec{r})]\sin\omega_0 t\cos\omega_0 t$$

Using the trigonometric identities $\cos^2\omega_0 t = \frac{1}{2} + \frac{1}{2}\cos(2\omega_0 t)$, $\sin^2\omega_0 t = \frac{1}{2} - \frac{1}{2}\cos(2\omega_0 t)$, and $\sin\omega_0 t\cos\omega_0 t = \frac{1}{2}\sin(2\omega_0 t)$, we write:

$$\vec{S}(\vec{r},t) = \frac{1}{2}[\vec{E}'(\vec{r})\times\vec{H}'(\vec{r}) + \vec{E}''(\vec{r})\times\vec{H}''(\vec{r})] + \frac{1}{2}[\vec{E}'(\vec{r})\times\vec{H}'(\vec{r}) - \vec{E}''(\vec{r})\times\vec{H}''(\vec{r})]\cos(2\omega_0 t)$$

$$+ \frac{1}{2}[\vec{E}'(\vec{r})\times\vec{H}''(\vec{r}) + \vec{E}''(\vec{r})\times\vec{H}'(\vec{r})]\sin(2\omega_0 t)$$

The time-averaged values of $\sin(2\omega_0 t)$ and $\cos(2\omega_0 t)$ are zero. Therefore,

$$\langle\vec{S}(\vec{r},t)\rangle = \frac{1}{2}[\vec{E}'(\vec{r})\times\vec{H}'(\vec{r}) + \vec{E}''(\vec{r})\times\vec{H}''(\vec{r})].$$

The above result is consistent with:

$$\langle\vec{S}(\vec{r},t)\rangle = \frac{1}{2}\text{Real}\{\vec{E}(\vec{r})\times\vec{H}^*(\vec{r})\} = \frac{1}{2}\text{Real}\{(\vec{E}'+i\vec{E}'')\times(\vec{H}'-i\vec{H}'')\}$$

$$= \frac{1}{2}[\vec{E}'(\vec{r})\times\vec{H}'(\vec{r}) + \vec{E}''(\vec{r})\times\vec{H}''(\vec{r})] \checkmark$$