Problem 11)

a) $E_x(\boldsymbol{r}_o,t)=\text{Re}\left\{\int_{\omega_1}^{\omega_2} E_o(\omega)\exp[-\omega n_I(\omega)z_o/c]\exp\{i[\omega n_R(\omega)z_o/c-\omega t]\}d\omega\right\}.$

Now, $\omega n_R(\omega)z_o/c-\omega t \approx [\omega_o n_R(\omega_o)z_o/c-\omega_o t]+\frac{d[\omega n_R(\omega)z_o/c-\omega t]}{d\omega}\Big|_{\omega=\omega_o}(\omega-\omega_o)$

$$=[\omega_o n_R(\omega_o)z_o/c-\omega_o t]+[n_R(\omega_o)z_o/c+\omega_o n'_R(\omega_o)z_o/c-t](\omega-\omega_o).$$

In the above equation $n'_R(\omega_o)=dn_R(\omega)/d\omega|_{\omega_o}$. We thus have

$$E_x(\boldsymbol{r}_o,t)\approx\text{Re}\Big\{\exp\{i[\omega_o n_R(\omega_o)z_o/c-\omega_o t]\}$$
$$\times\int_{\omega_1}^{\omega_2}\exp[-\omega n_I(\omega)z_o/c]E_o(\omega)\exp\{i\{[n_R(\omega_o)+\omega_o n'_R(\omega_o)](z_o/c)-t\}(\omega-\omega_o)\}d\omega\Big\}.$$

b) The last expression is similar to that in Problem (10), where the function under the integral sign varies slowly with time. The integral thus defines the envelope of the pulse, and the peak occurs where the integrand's phase-factor disappears. The peak of the envelope thus occurs at time $t=t_o$, where we have $[n_R(\omega_o)+\omega_o n'_R(\omega_o)](z_o/c)-t_o=0.$

c) The last expression in part (b) now yields the group velocity, $V_g=z_o/t_o$, as follows:

$$V_g=c/[n_R(\omega_o)+\omega_o n'_R(\omega_o)].$$

It is clear that only the real part $n_R(\omega)$ of the refractive index participates in the phase-factor under the integral sign. Ultimately, of course, it is the phase-factor that determines the group velocity and, therefore, the function $n_R(\omega)$ and its derivative $n'_R(\omega)$ determine V_g. In contrast, $n_I(\omega)$ does *not* affect the phase-factor; rather, it modifies the amplitude $E_o(\omega)$ of the various spectral components of the pulse. With increasing z_o, the attenuation coefficient $\exp[-\omega n_I(\omega)z_o/c]$, which modifies $E_o(\omega)$, becomes more and more prominent. Aside from an overall attenuation of the pulse, $n_I(\omega)$ also *distorts* the shape of the spectrum and, therefore, the shape of the pulse.