Problem 10)

$$f(t) = \mathrm{Re}\left\{\{A_\mathrm{o}\exp(\mathrm{i}\varphi_\mathrm{o}) + A_1\exp[\mathrm{i}(\varphi_1 + \Delta\omega t)] + A_2\exp[\mathrm{i}(\varphi_2 - \Delta\omega t)]\}\exp(-\mathrm{i}\omega_\mathrm{o} t)\right\}.$$

Defining $g(t) = A_\mathrm{o}\exp(\mathrm{i}\varphi_\mathrm{o}) + A_1\exp[\mathrm{i}(\varphi_1 + \Delta\omega t)] + A_2\exp[\mathrm{i}(\varphi_2 - \Delta\omega t)],$ we will have

$$f(t) = \mathrm{Re}[g(t)\exp(-\mathrm{i}\omega_\mathrm{o} t)].$$

In the complex plane, therefore, in consequence of multiplication with $\exp(-\mathrm{i}\omega_\mathrm{o} t)$, the entire function $g(t)$ rotates clockwise with frequency ω_o. In addition, $g(t)$ varies slowly with time because of the terms that contain $\Delta\omega t$. When the three complex vectors comprising $g(t)$ overlap, the magnitude of $g(t)$ will reach its maximum. This happens when

$$\varphi_1 + \Delta\omega t_\mathrm{o} = \varphi_2 - \Delta\omega t_\mathrm{o} = \varphi_\mathrm{o} \quad \rightarrow \quad t_\mathrm{o} = \Delta\varphi/\Delta\omega.$$

The process is periodic, of course, repeating itself with period T where $\Delta\omega T = 2\pi$, that is, $T = 2\pi/\Delta\omega$. We thus have $t_\mathrm{o} = mT + (\Delta\varphi/\Delta\omega) = (2m\pi + \Delta\varphi)/\Delta\omega$, with m being an arbitrary integer.