Problem 6) a) Particle m: $m x_m''(t) = -qE_{xo}\cos(\omega t) - \alpha[x_m(t) - x_M(t)] - \beta_m x_m'(t),$ (1a)

Particle M: $M x_M''(t) = +qE_{xo}\cos(\omega t) + \alpha[x_m(t) - x_M(t)] - \beta_M x_M'(t).$ (1b)

Adding Eq.(1a) to Eq.(1b), then using the fact of stationarity of the center-of-mass, namely, $m x_m(t) + M x_M(t) = 0$, leads to the trivial identity $0 = 0$. The two equations, therefore, have identical content, and one can derive all the desired information concerning the oscillator from either Eq.(1a) or Eq.(1b).

b) We choose for further analysis the equation of motion of the particle m, namely, Eq.(1a), then substitute for $x_m(t)$, $x_M(t)$, and $E(\boldsymbol{r}, t)$ the corresponding complex expressions to arrive at

$$-m\omega^2 x_{m0} + \alpha(x_{m0} - x_{M0}) - \mathrm{i}\omega\beta_m x_{m0} = -qE_{xo}. \quad (2)$$

Using the stationarity condition, $x_{M0} = -(m/M)x_{m0}$, the above equation can be written as

$$\omega^2 x_{m0} - (\alpha/m)[x_{m0} + (m/M)x_{m0}] + \mathrm{i}\omega(\beta_m/m)x_{m0} = (q/m)E_{xo}, \quad (3)$$

which yields the following solution for x_{m0}:

$$x_{m0} = (q/m)E_{xo}/[\omega^2 - \alpha(1/m + 1/M) + \mathrm{i}(\beta_m/m)\,\omega]. \quad (4)$$

Since the polarization density is given by $P_{xo} = Nq(x_{M0} - x_{m0}) = -Nq[(m/M)+1]x_{m0}$, we will have

$$\varepsilon_o C(\omega) = P_{xo}/E_{xo} = -\frac{Nq^2[(1/m)+(1/M)]}{\omega^2 - \alpha[(1/m)+(1/M)] + \mathrm{i}(\beta_m/m)\,\omega}\,. \quad (5)$$

c) From Eq.(5), the oscillator parameters are found to be

$$\omega_p^2 = (Nq^2/\varepsilon_o)[(1/m)+(1/M)], \quad (6a)$$

$$\omega_o^2 = \alpha[(1/m)+(1/M)], \quad (6b)$$

$$\gamma = \beta_m/m. \quad (6c)$$