Problem 2) As in Problem 1, the volume of interest is $\Delta d\Delta s\cos\theta$, of which one half is to the left of Δs and one half to the right. Here Δd is the change of $d(t)$ over a time interval Δt, that is, $\Delta d = d(t+\Delta t) - d(t)$. If a dipole is located within this volume, one of its charges (either $+q$ or $-q$) will cross the surface Δs during the time interval Δt. Irrespective of which charge crosses, the current flowing from left to right will be $q/\Delta t$. The current crossing Δs due to all dipoles that are oriented at an angle θ to $\Delta \boldsymbol{s}$ is thus given by

$$\Delta I(t) = (q/\Delta t)(\Delta d\,\Delta s\cos\theta)N(\theta)\,\mathrm{d}\theta = [\partial \boldsymbol{p}(t)/\partial t]\cdot\Delta\boldsymbol{s}\,N(\theta)\,\mathrm{d}\theta.$$

The corresponding current density, $\Delta\boldsymbol{J}(t) = [\partial\boldsymbol{p}(t)/\partial t]N(\theta)\mathrm{d}\theta$, integrated over all possible angles θ, then yields

$$\boldsymbol{J}(t) = \frac{\partial}{\partial t}\int_0^\pi \boldsymbol{p}(t)N(\theta)\,\mathrm{d}\theta = \frac{\partial \boldsymbol{P}(t)}{\partial t},$$

where $\boldsymbol{P}(t)$ is the local polarization density. We may thus write, quite generally, $\boldsymbol{J}(\boldsymbol{r},t) = \partial\boldsymbol{P}(\boldsymbol{r},t)/\partial t$.

Note that elongation of dipoles, i.e., change of dipole moment $\boldsymbol{p}$ through changes in d, is but one mechanism by which the polarization $\boldsymbol{P}$ of a given medium can become time-dependent. Other possible mechanisms include: charge transfer in time (i.e., q is time-dependent), rotation of dipoles (i.e., θ becomes time-dependent), and motion of entire dipoles through the surface element Δs. It can be shown that all these mechanisms, individually or in combination, make a contribution to the current-density $\boldsymbol{J}$ that can be expressed in the aforementioned form, namely, $\boldsymbol{J}(\boldsymbol{r},t) = \partial\boldsymbol{P}(\boldsymbol{r},t)/\partial t$.