Problem 1) The volume of interest is $d\Delta s\cos\theta$, of which one half is to the left and one half to the right of the surface element Δs. Any dipole whose center point is within this volume contributes a charge $+q$ to one side of the surface element and a charge $-q$ to the other side. The number of dipoles having the orientation angle θ within this volume is $(d\Delta s\cos\theta)N(\theta)\mathrm{d}\theta$, their contribution to charges accumulated on the left-hand-side being

$$\Delta Q = -q(d\Delta s\cos\theta)N(\theta)\mathrm{d}\theta = -\boldsymbol{p}\cdot\Delta\boldsymbol{s}\,N(\theta)\mathrm{d}\theta.$$

To find the charge Q contributed by all dipoles we must integrate over θ from 0 to π. Thus $Q = -(\int_0^{\pi}\boldsymbol{p}N(\theta)\mathrm{d}\theta)\cdot\Delta\boldsymbol{s} = -\boldsymbol{P}\cdot\Delta\boldsymbol{s}$, where $\boldsymbol{P}$ is the polarization density of the medium. (The charge accumulated on the right-hand side of the surface element Δs is $+\boldsymbol{P}\cdot\Delta\boldsymbol{s}$.) If we now consider a small cube and compute the total charge inside the cube it will be the sum of $-\boldsymbol{P}\cdot\Delta\boldsymbol{s}$ over all six surfaces of the cube. Normalization by the volume of the cube yields $\rho = -\boldsymbol{\nabla}\cdot\boldsymbol{P}$.