5-53)

a) $$\boldsymbol{B}(\boldsymbol{r}) = \boldsymbol{\nabla} \times \boldsymbol{A}(\boldsymbol{r}) = \frac{\partial(\sin\theta A_\phi)}{r\sin\theta\partial\theta}\hat{\boldsymbol{r}} - \frac{\partial(rA_\phi)}{r\partial r}\hat{\boldsymbol{\theta}}$$

$$= \begin{cases} \dfrac{\partial\left(M_0 r\sin^2\theta/3\right)}{r\sin\theta\partial\theta}\hat{\boldsymbol{r}} - \dfrac{\partial\left(M_0 r^2\sin\theta/3\right)}{r\partial r}\hat{\boldsymbol{\theta}}; & r < R, \\ \dfrac{\partial\left(M_0 R^3\sin^2\theta/3r^2\right)}{r\sin\theta\partial\theta}\hat{\boldsymbol{r}} - \dfrac{\partial\left(M_0 R^3\sin\theta/3r\right)}{r\partial r}\hat{\boldsymbol{\theta}}; & r > R. \end{cases}$$

$$= \begin{cases} \tfrac{2}{3}M_0(\cos\theta\,\hat{\boldsymbol{r}} - \sin\theta\,\hat{\boldsymbol{\theta}}); & r < R, \\ \tfrac{1}{3}M_0 R^3(2\cos\theta\,\hat{\boldsymbol{r}} + \sin\theta\,\hat{\boldsymbol{\theta}})/r^3; & r > R. \end{cases}$$

The B-field inside the sphere may be further simplified and written as $\boldsymbol{B}(\boldsymbol{r}) = \tfrac{2}{3}M_0\hat{\boldsymbol{z}}$.

b) $$\boldsymbol{J}_{\text{bound}}^{(e)} = \mu_0^{-1}\boldsymbol{\nabla} \times \boldsymbol{M}(\boldsymbol{r}) = \mu_0^{-1}\boldsymbol{\nabla} \times \left[M_0\,\text{Sphere}(r/R)(\cos\theta\,\hat{\boldsymbol{r}} - \sin\theta\,\hat{\boldsymbol{\theta}})\right]$$

$$= \mu_0^{-1}\left[\frac{\partial(rM_\theta)}{r\partial r} - \frac{\partial M_r}{r\partial\theta}\right]\hat{\boldsymbol{\phi}}$$

$$= \mu_0^{-1}M_0\left[-\frac{\partial[r\text{Sphere}(r/R)\sin\theta]}{r\partial r} - \frac{\partial[\text{Sphere}(r/R)\cos\theta]}{r\partial\theta}\right]\hat{\boldsymbol{\phi}}$$

$$= \mu_0^{-1}M_0\left[-\frac{[\text{Sphere}(r/R) - r\delta(r-R)]\sin\theta}{r} + \frac{\text{Sphere}(r/R)\sin\theta}{r}\right]\hat{\boldsymbol{\phi}}$$

$$= \mu_0^{-1}M_0\delta(r-R)\sin\theta\,\hat{\boldsymbol{\phi}}.$$

Because of the δ-function appearing in the above expression for current-density, the bound current is a *surface current*, having a density of $\boldsymbol{J}_s(r = R, \theta, \phi) = \mu_0^{-1}M_0\sin\theta\,\hat{\boldsymbol{\phi}}$. Note that the bound current circulates in the same direction, $\hat{\boldsymbol{\phi}}$, everywhere on the spherical surface. The current-density is at its maximum on the equator, and drops to zero at the poles of the sphere.

c) The perpendicular B-field at the surface of the sphere is $B_r = \tfrac{2}{3}M_0\cos\theta$, which is the same immediately below and immediately above the surface for each θ. The continuity of $B_\perp$ is thus established.

As for the tangential H-field, in the region immediately above the surface we have

$$H_\parallel(r = R^+, \theta, \phi) = \mu_0^{-1}B_\theta(r = R^+, \theta, \phi) = \tfrac{1}{3}\mu_0^{-1}M_0\sin\theta.$$

In the region immediately below the surface, the tangential component of the H-field is

$$H_\parallel(r = R^-, \theta, \phi) = \mu_0^{-1}[B_\theta(r = R^-, \theta, \phi) - M_\theta(r = R^-, \theta, \phi)]$$

$$= \mu_0^{-1}(-\tfrac{2}{3}M_0\sin\theta + M_0\sin\theta) = \tfrac{1}{3}\mu_0^{-1}M_0\sin\theta$$

Clearly, the tangential component of the H-field is continuous at the sphere surface. This is expected, considering that no free surface-current exists at the surface of the sphere. It is not difficult to see that the tangential B-field has a discontinuity of magnitude $M_0\sin\theta$, which is equal to the bound surface-current-density, found in part (b), multiplied by μ_0. This, of course, is consistent with Maxwell's second equation, $\boldsymbol{\nabla} \times \boldsymbol{B}(\boldsymbol{r}, t) = \mu_0\boldsymbol{J}_{\text{total}}^{(e)}(\boldsymbol{r}, t) + \mu_0\varepsilon_0\partial\boldsymbol{E}(\boldsymbol{r}, t)/\partial t$.