5-52) a) $\boldsymbol{J}(\boldsymbol{r},t) = I_0\delta(r_\parallel - R)\delta(z)\hat{\boldsymbol{\phi}}$. Note that each δ-function has units of 1/m. Therefore, the units of $\boldsymbol{J}(\boldsymbol{r},t)$ are ampere/m^2, as they should be.

b) In cylindrical coordinates, we have

$$\boldsymbol{\nabla}\cdot\boldsymbol{J}(\boldsymbol{r},t) = \frac{\partial(r_\parallel J_{r_\parallel})}{r_\parallel \partial r_\parallel} + \frac{\partial J_\phi}{r_\parallel \partial\phi} + \frac{\partial J_z}{\partial z} = \frac{\partial[I_0\delta(r_\parallel - R)\delta(z)]}{r_\parallel \partial\phi} = 0.$$

Consequently, $\partial\rho(\boldsymbol{r},t)/\partial t = 0$. This indicates that the charge-density can have any arbitrary distribution throughout space so long as it does *not* vary with time. Any constant (i.e., time-independent) charge distribution is therefore allowed.