Problem 5.51) a) Since the units of $\boldsymbol{M}(\boldsymbol{r},t)$ are weber/m^2 and the delta-function has units of 1/m, the coefficient M_{so} must have the units of weber/m.

b) In the absence of $\rho_{\text{free}}(\boldsymbol{r},t)$ and $\boldsymbol{P}(\boldsymbol{r},t)$, the bound electric charge-density of the magnetized sheet is zero, that is, $\rho_{\text{bound}}^{(e)}=0$, while the bound current-density is given by

$$\boldsymbol{J}_{\text{bound}}^{(e)}=\mu_o^{-1}\boldsymbol{\nabla}\times\boldsymbol{M}(\boldsymbol{r},t)=\mu_o^{-1}(\partial M_z/\partial y)\hat{\boldsymbol{x}}=\mu_o^{-1}M_{so}\delta'(y)\cos(\omega_o t)\hat{\boldsymbol{x}}.$$

c) Since the electric charge-density of the sheet is zero everywhere, we have $\psi(\boldsymbol{r},t)=0$. As for the vector potential, we use the symmetry of the problem and compute $\boldsymbol{A}(\boldsymbol{r},t)$ only at $(x=0,y,z=0)$, as follows:

$$\boldsymbol{A}(\boldsymbol{r},t)=(\mu_o/4\pi)\int_{-\infty}^{\infty}\frac{\boldsymbol{J}_{\text{bound}}^{(m)}(\boldsymbol{r}',t-|\boldsymbol{r}-\boldsymbol{r}'|/c)}{|\boldsymbol{r}-\boldsymbol{r}'|}\mathrm{d}\boldsymbol{r}'$$

$$=\frac{M_{so}\hat{\boldsymbol{x}}}{4\pi}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\frac{\delta'(y')\cos\left[\omega_o\left(t-\sqrt{x'^2+(y-y')^2+z'^2}/c\right)\right]}{\sqrt{x'^2+(y-y')^2+z'^2}}\mathrm{d}x'\mathrm{d}y'\mathrm{d}z'$$

$$=\frac{M_{so}\hat{\boldsymbol{x}}}{4\pi}\left\{\cos(\omega_o t)\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\delta'(y')\mathrm{d}y'\mathrm{d}x'\int_{-\infty}^{\infty}\frac{\cos\{(\omega_o/c)\sqrt{x'^2+(y-y')^2+z'^2}\}}{\sqrt{x'^2+(y-y')^2+z'^2}}\mathrm{d}z'\right.$$

$$\left.+\sin(\omega_o t)\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\delta'(y')\mathrm{d}y'\mathrm{d}x'\int_{-\infty}^{\infty}\frac{\sin\{(\omega_o/c)\sqrt{x'^2+(y-y')^2+z'^2}\}}{\sqrt{x'^2+(y-y')^2+z'^2}}\mathrm{d}z'\right\}$$

G&R 3.876-1,2 →

$$=\tfrac{1}{4}M_{so}\hat{\boldsymbol{x}}\left\{-\cos(\omega_o t)\int_{-\infty}^{\infty}\delta'(y')\mathrm{d}y'\int_{-\infty}^{\infty}Y_0[(\omega_o/c)\sqrt{x'^2+(y-y')^2}]\mathrm{d}x'\right.$$

$$\left.+\sin(\omega_o t)\int_{-\infty}^{\infty}\delta'(y')\mathrm{d}y'\int_{-\infty}^{\infty}J_0[(\omega_o/c)\sqrt{x'^2+(y-y')^2}]\mathrm{d}x'\right\}$$

G&R 6.677-3,4 →

$$=\tfrac{1}{2}M_{so}\hat{\boldsymbol{x}}\left\{-(\omega_o/c)^{-1}\cos(\omega_o t)\int_{-\infty}^{\infty}\delta'(y')\sin[(\omega_o/c)\sqrt{(y-y')^2}]\mathrm{d}y'\right.$$

$$\left.+(\omega_o/c)^{-1}\sin(\omega_o t)\int_{-\infty}^{\infty}\delta'(y')\cos[(\omega_o/c)\sqrt{(y-y')^2}]\mathrm{d}y'\right\}$$

$$=-\tfrac{1}{2}M_{so}\hat{\boldsymbol{x}}\,\mathrm{sign}(y)[\cos(\omega_o t)\cos(\omega_o|y|/c)+\sin(\omega_o t)\sin(\omega_o|y|/c)]$$

← Sifting property of $\delta'(\cdot)$

$$=-\tfrac{1}{2}M_{so}\,\mathrm{sign}(y)\cos[\omega_o(t-|y|/c)]\hat{\boldsymbol{x}}.$$

d) $$\boldsymbol{E}(\boldsymbol{r},t)=-\boldsymbol{\nabla}\psi-\partial\boldsymbol{A}/\partial t=-\tfrac{1}{2}\omega_o M_{so}\,\mathrm{sign}(y)\sin[\omega_o(t-|y|/c)]\hat{\boldsymbol{x}}.$$

$$\boldsymbol{B}(\boldsymbol{r},t)=\boldsymbol{\nabla}\times\boldsymbol{A}=-(\partial A_x/\partial y)\hat{\boldsymbol{z}}=\tfrac{1}{2}(\omega_o/c)M_{so}\sin[\omega_o(t-|y|/c)]\hat{\boldsymbol{z}}.$$