Problem 5.48) a) $J_{\text{bound}}^{(e)} = \mu_o^{-1}\boldsymbol{\nabla}\times\boldsymbol{M}(\boldsymbol{r},t) = \mu_o^{-1}\boldsymbol{\nabla}\times[m_o\delta(x)\delta(y)\delta(z)\hat{\boldsymbol{z}}]$

$$= \mu_o^{-1}m_o[\delta(x)\delta'(y)\hat{\boldsymbol{x}} - \delta'(x)\delta(y)\hat{\boldsymbol{y}}]\delta(z).$$

b) $\boldsymbol{A}(\boldsymbol{r}) = (\mu_o/4\pi)\int_{-\infty}^{\infty}[\boldsymbol{J}(\boldsymbol{r}')/|\boldsymbol{r}-\boldsymbol{r}'|]\mathrm{d}\boldsymbol{r}'$

$$= \frac{m_o}{4\pi}\int_{-\infty}^{\infty}\frac{[\delta(x')\delta'(y')\hat{\boldsymbol{x}} - \delta'(x')\delta(y')\hat{\boldsymbol{y}}]\delta(z')}{\sqrt{(x-x')^2+(y-y')^2+(z-z')^2}}\mathrm{d}x'\mathrm{d}y'\mathrm{d}z'$$ ← Sifting property of $\delta(\cdot)$

$$= \frac{m_o\hat{\boldsymbol{x}}}{4\pi}\int_{-\infty}^{\infty}\frac{\delta'(y')}{\sqrt{x^2+(y-y')^2+z^2}}\mathrm{d}y' - \frac{m_o\hat{\boldsymbol{y}}}{4\pi}\int_{-\infty}^{\infty}\frac{\delta'(x')}{\sqrt{(x-x')^2+y^2+z^2}}\mathrm{d}x'$$ ← Sifting property of $\delta'(\cdot)$

$$= -\frac{m_o\hat{\boldsymbol{x}}}{4\pi}\frac{y}{(x^2+y^2+z^2)^{3/2}} + \frac{m_o\hat{\boldsymbol{y}}}{4\pi}\frac{x}{(x^2+y^2+z^2)^{3/2}}$$

$$= \frac{m_o(x\hat{\boldsymbol{y}} - y\hat{\boldsymbol{x}})}{4\pi(x^2+y^2+z^2)^{3/2}} = \frac{m_o\hat{\boldsymbol{z}}\times\boldsymbol{r}}{4\pi r^3} = \frac{m_o[(\cos\theta)\hat{\boldsymbol{r}} - (\sin\theta)\hat{\boldsymbol{\theta}}]\times\hat{\boldsymbol{r}}}{4\pi r^2} = \frac{m_o\sin\theta}{4\pi r^2}\hat{\boldsymbol{\phi}}.$$

c) $\boldsymbol{B}(\boldsymbol{r}) = \boldsymbol{\nabla}\times\boldsymbol{A}(\boldsymbol{r}) = \boldsymbol{\nabla}\times\left(\frac{m_o\sin\theta}{4\pi r^2}\hat{\boldsymbol{\phi}}\right) = \frac{m_o}{4\pi}\left[\frac{\partial(\sin^2\theta/r^2)}{r\sin\theta\,\partial\theta}\hat{\boldsymbol{r}} - \frac{\partial(\sin\theta/r)}{r\partial r}\hat{\boldsymbol{\theta}}\right]$

$$= \frac{m_o}{4\pi}\left[\frac{2\cos\theta}{r^3}\hat{\boldsymbol{r}} + \frac{\sin\theta}{r^3}\hat{\boldsymbol{\theta}}\right] = \frac{m_o}{4\pi r^3}(2\cos\theta\,\hat{\boldsymbol{r}} + \sin\theta\,\hat{\boldsymbol{\theta}}).$$