Problem 5.46) a) Setting $\rho_{\text{free}} = 0$, $\boldsymbol{J}_{\text{free}} = 0$, $\boldsymbol{D} = \varepsilon_0\varepsilon\boldsymbol{E}$, and $\boldsymbol{B} = \mu_0\mu\boldsymbol{H}$, Maxwell's equations can be simplified as follows:

$$\boldsymbol{\nabla}\cdot\boldsymbol{D}(\boldsymbol{r},t) = \varepsilon_0\varepsilon\boldsymbol{\nabla}\cdot\boldsymbol{E}(\boldsymbol{r},t) = 0 \quad\rightarrow\quad \boldsymbol{\nabla}\cdot\boldsymbol{E}(\boldsymbol{r}) = 0.$$

$$\boldsymbol{\nabla}\times\boldsymbol{H}(\boldsymbol{r},t) = \partial\boldsymbol{D}(\boldsymbol{r},t)/\partial t = -\mathrm{i}\omega\varepsilon_0\varepsilon\boldsymbol{E}(\boldsymbol{r},t) \quad\rightarrow\quad \boldsymbol{\nabla}\times\boldsymbol{H}(\boldsymbol{r}) = -\mathrm{i}\omega\varepsilon_0\varepsilon\boldsymbol{E}(\boldsymbol{r}).$$

$$\boldsymbol{\nabla}\times\boldsymbol{E}(\boldsymbol{r},t) = -\,\partial\boldsymbol{B}(\boldsymbol{r},t)/\partial t = \mathrm{i}\omega\mu_0\mu\boldsymbol{H}(\boldsymbol{r},t) \quad\rightarrow\quad \boldsymbol{\nabla}\times\boldsymbol{E}(\boldsymbol{r}) = \mathrm{i}\omega\mu_0\mu\boldsymbol{H}(\boldsymbol{r}).$$

$$\boldsymbol{\nabla}\cdot\boldsymbol{B}(\boldsymbol{r},t) = \mu_0\mu\boldsymbol{\nabla}\cdot\boldsymbol{H}(\boldsymbol{r},t) = 0 \quad\rightarrow\quad \boldsymbol{\nabla}\cdot\boldsymbol{H}(\boldsymbol{r}) = 0.$$

b) Taking the curl of the 3rd equation, then substituting for $\boldsymbol{\nabla}\times\boldsymbol{H}$ from the 2nd equation, yields

$$\boldsymbol{\nabla}\times\boldsymbol{\nabla}\times\boldsymbol{E}(\boldsymbol{r}) = \mathrm{i}\omega\mu_0\mu\boldsymbol{\nabla}\times\boldsymbol{H}(\boldsymbol{r}) = \mu_0\varepsilon_0\mu\varepsilon\omega^2\boldsymbol{E}(\boldsymbol{r}) = (\sqrt{\mu\varepsilon}\,\omega/c)^2\boldsymbol{E}(\boldsymbol{r}).$$

From the 1st equation, we know that $\boldsymbol{\nabla}\cdot\boldsymbol{E} = 0$. Therefore,

$$\boldsymbol{\nabla}[\boldsymbol{\nabla}\cdot\boldsymbol{E}(\boldsymbol{r})] - \boldsymbol{\nabla}^2\boldsymbol{E}(\boldsymbol{r}) = (\sqrt{\mu\varepsilon}\,\omega/c)^2\boldsymbol{E}(\boldsymbol{r}) \quad\rightarrow\quad \boldsymbol{\nabla}^2\boldsymbol{E}(\boldsymbol{r}) + (\sqrt{\mu\varepsilon}\,\omega/c)^2\boldsymbol{E}(\boldsymbol{r}) = 0.$$

c) Taking the curl of the 2nd equation, then substituting for $\boldsymbol{\nabla}\times\boldsymbol{E}$ from the 3rd equation, yields

$$\boldsymbol{\nabla}\times\boldsymbol{\nabla}\times\boldsymbol{H}(\boldsymbol{r}) = -\mathrm{i}\omega\varepsilon_0\varepsilon\boldsymbol{\nabla}\times\boldsymbol{E}(\boldsymbol{r}) = \mu_0\varepsilon_0\mu\varepsilon\omega^2\boldsymbol{H}(\boldsymbol{r}) = (\sqrt{\mu\varepsilon}\,\omega/c)^2\boldsymbol{H}(\boldsymbol{r}).$$

From the 4th equation, we know that $\boldsymbol{\nabla}\cdot\boldsymbol{H} = 0$. Therefore,

$$\boldsymbol{\nabla}[\boldsymbol{\nabla}\cdot\boldsymbol{H}(\boldsymbol{r})] - \boldsymbol{\nabla}^2\boldsymbol{H}(\boldsymbol{r}) = (\sqrt{\mu\varepsilon}\,\omega/c)^2\boldsymbol{H}(\boldsymbol{r}) \quad\rightarrow\quad \boldsymbol{\nabla}^2\boldsymbol{H}(\boldsymbol{r}) + (\sqrt{\mu\varepsilon}\,\omega/c)^2\boldsymbol{H}(\boldsymbol{r}) = 0.$$