Problem 39)
$$\vec{A}(\vec{r},t)=\frac{\mu_0\hat{z}}{4\pi}\iint_{-\infty}^{\infty}\frac{J_{so}\sin[\omega(t-|\vec{r}-\vec{r}'|/c)-ky']}{|\vec{r}-\vec{r}'|}\,dy'dz',$$

where $|\vec{r}-\vec{r}'|=\sqrt{x^2+(y-y')^2+z'^2}$. A Change of variable $u'=y'-y$ yields:

$$\vec{A}(\vec{r},t)=\frac{\mu_0 J_{so}\hat{z}}{4\pi}\iint_{-\infty}^{\infty}\frac{\sin(\omega t-ky-ku'-k_0\sqrt{x^2+u'^2+z'^2})}{\sqrt{x^2+u'^2+z'^2}}\,du'dz'$$

$$=\frac{\mu_0 J_{so}\hat{z}}{4\pi}\left\{\int_{-\infty}^{\infty}\sin(\omega t-ky-ku')\left[\int_{-\infty}^{\infty}\frac{\cos k_0\sqrt{x^2+u'^2+z'^2}}{\sqrt{x^2+u'^2+z'^2}}dz'\right]du'\right.$$
$$\left.-\int_{-\infty}^{\infty}\cos(\omega t-ky-ku')\left[\int_{-\infty}^{\infty}\frac{\sin k_0\sqrt{x^2+u'^2+z'^2}}{\sqrt{x^2+u'^2+z'^2}}dz'\right]du'\right\}\Rightarrow$$

$$\vec{A}(\vec{r},t)=-\frac{\mu_0 J_{so}\hat{z}}{4}\left\{\int_{-\infty}^{\infty}\sin(\omega t-ky-ku')Y_0(k_0\sqrt{x^2+u'^2})\,du'\right.$$
$$\left.+\int_{-\infty}^{\infty}\cos(\omega t-ky-ku')J_0(k_0\sqrt{x^2+u'^2})\,du'\right\}$$

$$=-\frac{\mu_0 J_{so}\hat{z}}{4}\left\{\sin(\omega t-ky)\int_{-\infty}^{\infty}\cos(ku')Y_0(k_0\sqrt{x^2+u'^2})\,du'-\cos(\omega t-ky)\int_{-\infty}^{\infty}\sin(ku')Y_0(k_0\sqrt{x^2+u'^2})\,du'\right.$$
$$\left.+\cos(\omega t-ky)\int_{-\infty}^{\infty}\cos(ku')J_0(k_0\sqrt{x^2+u'^2})\,du'+\sin(\omega t-ky)\int_{-\infty}^{\infty}\sin(ku')J_0(k_0\sqrt{x^2+u'^2})\,du'\right\}$$

The integrals that contain $\sin(ku')$ are zero because $\sin(ku')$ is an odd function of u', while $Y_0(\cdot)$ and $J_0(\cdot)$ are even. The other two integrals exist, and we have:

$$\vec{A}(\vec{r},t)=-\tfrac{1}{2}\mu_0 J_{so}\hat{z}\left\{\left[\int_0^{\infty}Y_0(k_0\sqrt{x^2+u'^2})\cos(ku')\,du'\right]\sin(\omega t-ky)\right.$$
$$\left.+\left[\int_0^{\infty}J_0(k_0\sqrt{x^2+u'^2})\cos(ku')\,du'\right]\cos(\omega t-ky)\right\}\Rightarrow$$

$$\vec{A}(\vec{r},t) = \begin{cases} -\dfrac{\mu_0 J_{s0}\hat{z}}{2\sqrt{k_0^2-K^2}}\left\{\sin(\sqrt{k_0^2-K^2}\,x)\sin(\omega t-Ky) + \cos(\sqrt{k_0^2-K^2}\,x)\cos(\omega t-Ky)\right\}; & k_0>K \\ \dfrac{\mu_0 J_{s0}\hat{z}}{2\sqrt{K^2-k_0^2}}\exp(-\sqrt{K^2-k_0^2}\,x)\sin(\omega t-Ky); & k_0<K \end{cases}$$

$$\Rightarrow \vec{A}(\vec{r},t) = \begin{cases} -\dfrac{\mu_0 J_{s0}\hat{z}}{2\sqrt{k_0^2-K^2}}\cos(\omega t-Ky-\sqrt{k_0^2-K^2}\,x); & k_0>K \\ \dfrac{\mu_0 J_{s0}\hat{z}}{2\sqrt{K^2-k_0^2}}\exp(-\sqrt{K^2-k_0^2}\,x)\sin(\omega t-Ky); & k_0<K \end{cases}$$

$$\vec{E}(\vec{r},t) = -\frac{\partial \vec{A}}{\partial t} = \begin{cases} -\dfrac{Z_0 J_{s0} k_0\hat{z}}{2\sqrt{k_0^2-K^2}}\sin(\omega t-Ky-\sqrt{k_0^2-K^2}\,x); & k_0>K \\ -\dfrac{Z_0 J_{s0} k_0\hat{z}}{2\sqrt{K^2-k_0^2}}\exp(-\sqrt{K^2-k_0^2}\,x)\cos(\omega t-Ky); & k_0<K \end{cases}$$

$$\vec{B}(\vec{r},t) = \vec{\nabla}\times\vec{A} = \frac{\partial A_z}{\partial y}\hat{x} - \frac{\partial A_z}{\partial x}\hat{y} \Rightarrow$$

$$\vec{B}(\vec{r},t) = \begin{cases} -\dfrac{\mu_0 J_{s0}}{2\sqrt{k_0^2-K^2}}(K\hat{x}-\sqrt{k_0^2-K^2}\,\hat{y})\sin(\omega t-Ky-\sqrt{k_0^2-K^2}\,x); & k_0>K \\ \dfrac{-\mu_0 J_{s0}}{2\sqrt{K^2-k_0^2}}[K\cos(\omega t-Ky)\hat{x} - \sqrt{K^2-k_0^2}\sin(\omega t-Ky)\hat{y}]\exp(-\sqrt{K^2-k_0^2}\,x); & k_0<K \end{cases}$$

The above formulas for $\vec{E}(\vec{r},t)$ and $\vec{B}(\vec{r},t)$ represent a plane-wave that, in the case of $k_0>K$, is propagating along a k-vector in the xy-plane, whereas, in the case of $k_0<K$, is evanescent. If we consider both sides of the current sheet, i.e., the region where $x>0$ and the region where $x<0$, then we'll find that $\vec{E}(x=0,y,z,t)$ is continuous across the current sheet, as is B_x. However, H_y is discontinuous at the sheet, with the magnitude of discontinuity being equal to surface current density $J_s(\vec{r},t)$.