Problem 37)

a) On the cylindrical walls of the cavity, where $\rho = R$, the tangential E-field (E_z in the present case) must vanish. Therefore, $J_0(R\omega_o/c) = 0$. Acceptable values of R are thus $R_n = cx_n/\omega_o$.

b) $$\nabla \times \boldsymbol{E}(\boldsymbol{r},t) = -\partial \boldsymbol{B}(\boldsymbol{r},t)/\partial t \quad \rightarrow \quad -(\partial E_z/\partial\rho)\hat{\boldsymbol{\phi}} = -\mu_o \partial \boldsymbol{H}(\boldsymbol{r},t)/\partial t$$

$$\rightarrow \partial \boldsymbol{H}(\boldsymbol{r},t)/\partial t = \mu_o^{-1} E_o\,(\omega_o/c)\,J_0'(\rho\omega_o/c)\cos(\omega_o t)\hat{\boldsymbol{\phi}} = -(\omega_o E_o/Z_o)\,J_1(\rho\omega_o/c)\cos(\omega_o t)\hat{\boldsymbol{\phi}}$$

$$\rightarrow \boldsymbol{H}(\boldsymbol{r},t) = -(E_o/Z_o)\,J_1(\rho\omega_o/c)\sin(\omega_o t)\hat{\boldsymbol{\phi}}.$$

c) Maxwell's 1st equation: $\nabla \cdot \boldsymbol{E} = 0 \quad \rightarrow \quad \partial E_z/\partial z = 0.$ Checks.

Maxwell's 2nd equation: $\nabla \times \boldsymbol{H} = \partial \boldsymbol{D}/\partial t \quad \rightarrow \quad \dfrac{1}{\rho}\dfrac{\partial(\rho H_\phi)}{\partial\rho}\hat{\boldsymbol{z}} = -\varepsilon_o\omega_o E_o J_0(\rho\omega_o/c)\sin(\omega_o t)\hat{\boldsymbol{z}}$

$$\rightarrow \quad -(E_o/Z_o)[(1/\rho)\,J_1(\rho\omega_o/c) + (\omega_o/c)\,J_1'(\rho\omega_o/c)]\sin(\omega_o t)\hat{\boldsymbol{z}} = -\varepsilon_o\omega_o E_o J_0(\rho\omega_o/c)\sin(\omega_o t)\hat{\boldsymbol{z}}$$

$$\rightarrow \quad -(E_o/Z_o)(\omega_o/c)\,J_0(\rho\omega_o/c)\sin(\omega_o t)\hat{\boldsymbol{z}} = -\varepsilon_o\omega_o E_o J_0(\rho\omega_o/c)\sin(\omega_o t)\hat{\boldsymbol{z}}.$$ Checks.

↑ Chapter 3, Eq.(41) ↑ $1/(Z_o c) = \varepsilon_o$

Maxwell's 4th equation: $\nabla \cdot \boldsymbol{H} = 0 \quad \rightarrow \quad (1/\rho)\,\partial H_\phi/\partial\phi = 0.$ Checks.

d) On the cylindrical surface of the cavity $E_r = 0$; no surface-charges therefore reside on this wall, that is, $\sigma_s(\rho = R, \phi, z, t) = 0$. The tangential H-field, however, is non-zero, yielding the following surface-current-density: $\boldsymbol{J}_s(\rho = R,\phi,z,t) = -H_\phi(\rho = R,\phi,z,t)\hat{\boldsymbol{z}} = (E_o/Z_o)\,J_1(R\omega_o/c)\sin(\omega_o t)\hat{\boldsymbol{z}}$.

At the top and bottom surfaces, the perpendicular D-field is $\varepsilon_o E_z(\rho,\phi,z = \pm L/2, t)\hat{\boldsymbol{z}}$. The surface-charge-density is thus given by $\sigma_s(\rho,\phi,z = \pm L/2,t) = \mp\varepsilon_o E_o J_0(\rho\omega_o/c)\cos(\omega_o t)$. Similarly, the surface-current-density is related to the tangential component of the H-field, as follows: $\boldsymbol{J}_s(\rho,\phi,z = \pm L/2,t) = \pm H_\phi(\rho,\phi,z = \pm L/2,t)\hat{\boldsymbol{\rho}} = \mp(E_o/Z_o)\,J_1(\rho\omega_o/c)\sin(\omega_o t)\hat{\boldsymbol{\rho}}$.

e) On the cylindrical surface, $\nabla\cdot\boldsymbol{J}_s = 0$ and $\sigma_s = 0$; therefore, the continuity equation is satisfied. Also, at the top and bottom flat surfaces we have

$$\nabla\cdot\boldsymbol{J}_s = \frac{1}{\rho}\frac{\partial(\rho J_{s\rho})}{\partial\rho} = \mp(E_o/Z_o)\,[(1/\rho)\,J_1(\rho\omega_o/c) + (\omega_o/c)\,J_1'(\rho\omega_o/c)]\sin(\omega_o t)$$

$$= \mp\varepsilon_o\omega_o E_o J_0(\rho\omega_o/c)\sin(\omega_o t),$$

$$\partial\sigma_s/\partial t = \pm\varepsilon_o\omega_o E_o J_0(\rho\omega_o/c)\sin(\omega_o t).$$

Clearly, $\nabla\cdot\boldsymbol{J}_s(\boldsymbol{r},t) + \partial\sigma_s(\boldsymbol{r},t)/\partial t = 0$ on the flat surfaces as well.