Solutions **Opti 501**

Problem 34)

a) $J_0(\sqrt{k_0^2-K^2}R)=0 \Rightarrow \sqrt{k_0^2-K^2}R$ must be a zero of the Bessel function $J_0(x)$. Under these circumstances, both $\vec{A}(\vec{r},t)$ and $\psi(\vec{r},t)$ within the metallic walls (indeed throughout the entire region $\rho>R$) vanish, resulting in zero values for both $\vec{E}(\vec{r},t)$ and $\vec{H}(\vec{r},t)$ in the region $\rho>R$. Inside the hollow core of the cylinder we'll have:

$$\psi(\vec{r},t)=-\tfrac{\pi}{2}(K/k_0)Z_0 J_{s0}R\,Y_0(\sqrt{k_0^2-K^2}R)\,J_0(\sqrt{k_0^2-K^2}\rho)\sin(\omega t-Kz);\quad \rho<R$$

$$\vec{A}(\vec{r},t)=-\tfrac{\pi}{2}\mu_0 J_{s0}R\,Y_0(\sqrt{k_0^2-K^2}R)\,J_0(\sqrt{k_0^2-K^2}\rho)\sin(\omega t-Kz)\,\hat{z};\quad \rho<R$$

b) $\mu_0\vec{H}(\vec{r},t)=\vec{\nabla}\times\vec{A}(\vec{r},t)=-\dfrac{\partial A_z}{\partial\rho}\hat{\phi}=\dfrac{-\pi}{2}\mu_0 J_{s0}R\sqrt{k_0^2-K^2}\,Y_0(\sqrt{\cdots}R)J_1(\sqrt{\cdots}\rho)\sin(\omega t-Kz)\hat{\phi}$

$\Rightarrow \vec{H}(\rho=R,\phi,z,t)=\dfrac{-\pi}{2}J_{s0}R\sqrt{k_0^2-K^2}\,Y_0(\sqrt{k_0^2-K^2}R)J_1(\sqrt{k_0^2-K^2}R)\sin(\omega t-Kz)\hat{\phi}$

Now, $Y_0(x)J_1(x)-J_0(x)Y_1(x)=\dfrac{2}{\pi x}$. At $x=\sqrt{k_0^2-K^2}R$, we have $J_0(x)=0$; therefore $Y_0(\sqrt{\cdots}R)J_1(\sqrt{\cdots}R)=\dfrac{2}{\pi\sqrt{k_0^2-K^2}R}$. Consequently:

$\vec{H}(\rho=R,\phi,z,t)=-J_{s0}\sin(\omega t-Kz)\hat{\phi}$. This is consistent with the surface current $\vec{J}_s(\rho=R,\phi,z,t)$, both in magnitude and direction ($H_\phi \perp J_z$).

$$\vec{E}(\vec{r},t)=-\vec{\nabla}\psi-\frac{\partial\vec{A}}{\partial t}=-\frac{\partial\psi}{\partial\rho}\hat{\rho}-\left(\frac{\partial\psi}{\partial z}+\frac{\partial A_z}{\partial t}\right)\hat{z}$$

$$=-\tfrac{\pi}{2}(K/k_0)Z_0J_{s0}R\sqrt{k_0^2-K^2}\,Y_0(\sqrt{\cdots}R)\,J_1(\sqrt{\cdots}\rho)\sin(\omega t-Kz)\hat{\rho}$$

$$-\left\{\tfrac{\pi}{2}(K^2/k_0)Z_0J_{s0}R\,Y_0(\sqrt{\cdots}R)J_0(\sqrt{\cdots}\rho)\cos(\omega t-Kz)-\tfrac{\pi}{2}\mu_0\omega J_{s0}R\,Y_0(\sqrt{\cdots}R)J_0(\sqrt{\cdots}\rho)\cos(\cdots)\right\}\hat{z}$$

$$\Rightarrow E_z(\vec{r},t)=\frac{\pi}{2}Z_0J_{s0}Rk_0\left(1-\frac{K^2}{k_0^2}\right)Y_0(\sqrt{k_0^2-K^2}R)\,J_0(\sqrt{k_0^2-K^2}\rho)\cos(\omega t-Kz)$$

At $\rho=R$ the E_z-field goes to zero because $J_0(\sqrt{k_0^2-K^2}R)$ is zero at the cylinder surface.

c) $\sigma_s(\rho=R,\phi,z,t) = -\epsilon_0 E_\rho(R,\phi,z,t) = \frac{\pi}{2}\epsilon_0(\kappa/k_0)Z_0 J_{s0} R\sqrt{\cdots}\,Y_0(\sqrt{\cdots}R)\,J_1(\sqrt{\cdots}R)\sin(\cdots)$

$$= \frac{\pi}{2}(\kappa/\omega)J_{s0}R\sqrt{k_0^2-\kappa^2}\,Y_0\left(\sqrt{k_0^2-\kappa^2}\,R\right)J_1\left(\sqrt{k_0^2-\kappa^2}\,R\right)\sin(\omega t-\kappa z)$$

As before, $Y_0(\sqrt{\cdots}\,R)\,J_1(\sqrt{\cdots}\,R) = \frac{2}{\pi\sqrt{k_0^2-\kappa^2}\,R}$. Therefore,

$$\sigma_s(R,\phi,z,t) = (\kappa/\omega)J_{s0}\sin(\omega t-\kappa z)$$

This is consistent with the conservation of charge equation $\vec{\nabla}\cdot\vec{J}_s + \frac{\partial\sigma_s}{\partial t} = 0$.