a) Limiting form of Bessel functions for small arguments: $J_0(x) \sim 1$, $Y_0(x) \sim \frac{2}{\pi} \ln x$. Therefore, in the near-field where $k_0 \rho \ll 1$, we have:

$$\vec{A}(\vec{r},t) \simeq -\frac{1}{4}\mu_0 I_0 \left\{ \frac{2}{\pi} \ln(k_0\rho) \sin\omega t + \cos\omega t \right\} \hat{z}$$

b) $\vec{B} = \vec{\nabla} \times \vec{A} = -\left(\frac{\partial}{\partial \rho} A_z\right)\hat{\phi}$ ← in Cylindrical Coordinates

$$\Rightarrow \mu_0 \vec{H} = \frac{1}{4}\mu_0 I_0 \frac{\partial}{\partial \rho}\left\{ \frac{2}{\pi} \ln(k_0\rho)\sin\omega t + \cos\omega t \right\}\hat{\phi} = \frac{\mu_0 I_0}{2\pi}\frac{\partial}{\partial \rho}\left[\ln(k_0\rho)\right]\sin\omega t\,\hat{\phi}$$

$$\Rightarrow \vec{H}(\vec{r},t) = \frac{I_0}{2\pi\rho}\sin\omega t\,\hat{\phi}$$

Ampère's law: $\vec{\nabla} \times \vec{H} = \vec{J} \Rightarrow \oint \vec{H} \cdot d\vec{\ell} = I \Rightarrow 2\pi\rho H_\phi = I(t) \Rightarrow I_0 \sin\omega t = I_0 \sin\omega t$ ✓

c) Since there are no charges in this problem, $\psi(\vec{r},t)=0$. Therefore,

$$\vec{E}(\vec{r},t) = -\frac{\partial \vec{A}}{\partial t} = \frac{1}{4}\mu_0 I_0 \left\{ \frac{2\omega}{\pi} \ln(k_0\rho)\cos\omega t - \omega \sin\omega t \right\} \hat{z}$$

Replacing ω with ck_0 and $\mu_0 c$ with Z_0, we find:

$$\vec{E}(\vec{r},t) = \frac{1}{4} k_0 Z_0 I_0 \left\{ \frac{2}{\pi} \ln(k_0\rho)\cos\omega t - \sin\omega t \right\} \hat{z}.$$