Solutions **Opti 501**

Problem 27)

$$\vec{A}(\vec{r},t)=\frac{\mu_0}{4\pi}\int_{V'}\frac{\vec{J}(\vec{r}\,',t-|\vec{r}-\vec{r}\,'|/c)}{|\vec{r}-\vec{r}\,'|}\,dv'$$

Let the cross-sectional area of the wire be s; then $\Delta v' = sd$. At the origin $\vec{r}\,'=0 \Rightarrow |\vec{r}-\vec{r}\,'|=|\vec{r}|=r$. Therefore, $\vec{J}(\vec{r}\,';t-|r-r'|/c)\,\Delta v'$ $\simeq \hat{z}\,J(0,t-r/c)\,sd = \hat{z}\,I(0,t-r/c)\,d = I_0 d \sin[2\pi f(t-r/c)]\,\hat{z}$.

Consequently, $\vec{A}(\vec{r},t)=\frac{\mu_0 I_0 d}{4\pi r}\sin[2\pi f(t-r/c)]\,\hat{z}$.

$$\vec{B}(\vec{r},t)=\vec{\nabla}\times\vec{A}(\vec{r},t)=(\partial A_z/\partial y)\,\hat{x}-(\partial A_z/\partial x)\,\hat{y}$$

$$=\frac{\mu_0 I_0 d}{4\pi}\left(\frac{\partial}{\partial y}\hat{x}-\frac{\partial}{\partial x}\hat{y}\right)\left\{(x^2+y^2+z^2)^{-1/2}\sin\left[2\pi f\left(t-\frac{1}{c}\sqrt{x^2+y^2+z^2}\right)\right]\right\}$$

$$=\frac{\mu_0 I_0 d}{4\pi}\left\{\left[-\frac{y}{r^3}\sin(\cdots)-\frac{2\pi f}{c}\frac{y}{r^2}\cos(\cdots)\right]\hat{x}+\left[\frac{x}{r^3}\sin(\cdots)+\frac{2\pi f}{c}\frac{x}{r^2}\cos(\cdots)\right]\hat{y}\right\}$$

Because of symmetry, we can confine our attention to the xz-plane where $\vec{r}=(x,0,z)$ and $\hat{y}=\hat{\phi}$ in spherical coordinates. Setting $y=0$ and $x=r\sin\theta$ in the above equation yields:

$$\vec{B}(\vec{r},t)=\frac{\mu_0 I_0 d}{4\pi}\sin\theta\left\{\frac{1}{r^2}\sin[2\pi f(t-r/c)]+\left(\frac{2\pi}{\lambda_0}\right)\frac{1}{r}\cos[2\pi f(t-r/c)]\right\}\hat{\phi}$$

where $\lambda_0 = c/f$ is the wavelength of the radiated field. Although we used Cartesian coordinates in deriving the above formula, the final result is expressed in spherical coordinates. Note that the first term drops as $1/r^2$, but the second term (the radiation or far field) drops only as $1/r$.