26)

a) $$\boxed{\vec{A}(\vec{r},t) = \frac{\mu_0 I_0 d}{4\pi r}\sin[\omega(t - r/c)]\hat{z}}$$

b) $$\vec{\nabla}\cdot\vec{A} + \frac{1}{c^2}\frac{\partial\psi}{\partial t} = 0 \Rightarrow \frac{\partial\psi}{\partial t} = -c^2\,\vec{\nabla}\cdot\vec{A} = -c^2\frac{\partial A_z}{\partial z} \Rightarrow$$

$$\frac{\partial\psi}{\partial t} = -\frac{1}{\cancel{\mu_0}\epsilon_0}\frac{\cancel{\mu_0} I_0 d}{4\pi}\frac{\partial}{\partial z}\left\{\frac{\sin(\omega t - k_0 r)}{r}\right\}, \quad \text{where } r = (x^2+y^2+z^2)^{1/2}.$$

$$\frac{\partial\psi}{\partial t} = -\frac{I_0 d}{4\pi\epsilon_0}\left\{-z(x^2+y^2+z^2)^{-3/2}\sin(\omega t - k_0 r) - \frac{k_0}{r}z(x^2+y^2+z^2)^{-1/2}\cos(\omega t - k_0 r)\right\}$$

$$= \frac{I_0 d}{4\pi\epsilon_0}\left\{\frac{z}{r^3}\sin(\omega t - k_0 r) + k_0\frac{z}{r^2}\cos(\omega t - k_0 r)\right\}$$

$$= \frac{I_0 d\cos\theta}{4\pi\epsilon_0}\left[\frac{1}{r^2}\sin(\omega t - k_0 r) + \frac{k_0}{r}\cos(\omega t - k_0 r)\right\} \leftarrow \cos\theta \text{ is substituted for } z/r.$$

$$\psi(\vec{r},t) = \int\frac{\partial\psi}{\partial t}dt = \frac{I_0 d\cos\theta}{4\pi\epsilon_0}\left[\frac{-1}{\omega r^2}\cos(\omega t - k_0 r) + \frac{k_0}{r\omega}\sin(\omega t - k_0 r)\right]$$

Since $\omega = ck_0$ and $c = 1/\sqrt{\mu_0\epsilon_0}$, we'll have:

$$\boxed{\psi(\vec{r},t) = \frac{Z_0 I_0 d}{4\pi}\cos\theta\left[\frac{1}{r}\sin(\omega t - k_0 r) - \frac{(\lambda_0/2\pi)}{r^2}\cos(\omega t - k_0 r)\right],}$$

which is the same result as obtained in the class.