a) Surface area $= 4\pi R^2 \Rightarrow Q = 4\pi R^2 \sigma_0$

b) Velocity of sphere's surface at $\vec{r}'$: $\vec{V}(\vec{r}') = (R\sin\theta')\omega\,\hat{\phi}$.

$$\Rightarrow \vec{J}_s(\vec{r}') = \sigma_0 \vec{V}(\vec{r}') = R\sin\theta'\,\omega\,\sigma_0\,\hat{\phi}$$

c) $$|\vec{r}-\vec{r}'|^2 = (x-x')^2 + (y-y')^2 + (z-z')^2 = (0 - R\sin\theta'\cos\phi')^2 +$$

$$(y - R\sin\theta'\sin\phi')^2 + (z - R\cos\theta')^2 = \cancel{R^2\sin^2\theta'\cos^2\phi'} + y^2 + \cancel{R^2\sin^2\theta'\sin^2\phi'}$$

$$-2Ry\sin\theta'\sin\phi' + z^2 + \cancel{R^2\cos^2\theta'} - 2Rz\cos\theta' = R^2 + y^2 + z^2 - 2Ry\sin\theta'\sin\phi' - 2Rz\cos\theta'$$

$$\Rightarrow |\vec r - \vec r\,'| = \sqrt{(y^2+z^2+R^2) - 2Ry\sin\theta'\sin\phi' - 2Rz\cos\theta'}$$

$$1/|\vec r-\vec r\,'| = \left[(y^2+z^2+R^2)\left(1-2\,\frac{Ry\sin\theta'\sin\phi'+Rz\cos\theta'}{y^2+z^2+R^2}\right)\right]^{-1/2}$$

$$\simeq (y^2+z^2+R^2)^{-1/2}\left[1+\frac{Ry\sin\theta'\sin\phi'+Rz\cos\theta'}{y^2+z^2+R^2}\right]$$

$$\simeq \frac{1}{\sqrt{y^2+z^2}} + \frac{Ry}{(y^2+z^2)^{3/2}}\sin\theta'\sin\phi' + \frac{Rz}{(y^2+z^2)^{3/2}}\cos\theta'$$

d) $$A_\phi(\vec r)\hat\phi = \frac{\mu_0}{4\pi}\int_{V'}\frac{\vec J(\vec r\,')\,dv'}{|\vec r-\vec r\,'|} = \frac{\mu_0}{4\pi}\int_{\theta'=0}^{\pi}\int_{\phi'=0}^{2\pi}\frac{J_s(\vec r\,')\sin\phi'\,\hat\phi}{|\vec r-\vec r\,'|}R^2\sin\theta'\,d\theta'\,d\phi'$$

$$\Rightarrow A_\phi(\vec r) = \frac{\mu_0}{4\pi}R^3\omega\sigma_0\int_{\theta'=0}^{\pi}\int_{\phi'=0}^{2\pi}|\vec r-\vec r\,'|^{-1}\sin\phi'\sin^2\theta'\,d\theta'\,d\phi'$$

e) Using $r=|\vec r|=\sqrt{y^2+z^2}$, $y=r\sin\theta$, $z=r\cos\theta$, we can write:

$1/|\vec r-\vec r\,'| \simeq \frac{1}{r} + \frac{R\sin\theta}{r^2}\sin\theta'\sin\phi' + \frac{R\cos\theta}{r^2}\cos\theta'$. Therefore,

$$A_\phi(\vec r) \cong \frac{\mu_0}{(4\pi)^2}(4\pi R^2\sigma_0)(R\omega)\left\{\frac{1}{r}\int_{\theta'=0}^{\pi}\sin^2\theta'\,d\theta'\cancelto{0}{\int_{\phi'=0}^{2\pi}\sin\phi'\,d\phi'} + \frac{R\sin\theta}{r^2}\int_{\theta'=0}^{\pi}\sin^3\theta'\,d\theta'\int_{\phi'=0}^{2\pi}\sin^2\phi'\,d\phi'\right.$$

$$\left. + \frac{R\cos\theta}{r^2}\int_{\theta'=0}^{\pi}\cos\theta'\sin^2\theta'\,d\theta'\cancelto{0}{\int_{\phi'=0}^{2\pi}\sin\phi'\,d\phi'}\right\}$$

$$\Rightarrow A_\phi(\vec r) \cong \frac{\mu_0}{12\pi}Q(R^2\omega)\frac{\sin\theta}{r^2} \Rightarrow \vec A(\vec r) \cong \frac{\mu_0}{4\pi r^2}\left(\frac{1}{3}QR^2\omega\right)(\hat z\times\hat r);\quad r \gg R$$

f) Comparison with $\vec A(\vec r)$ for small current loop reveals that $\vec m = \frac{1}{3}QR^2\omega\hat z$

g) mass-density (per unit surface area) = $\frac{M}{4\pi R^2}$

Density of linear momentum at $\vec{r}' = (M/4\pi R^2)\vec{V}(\vec{r}') = (\frac{M}{4\pi R})\sin\theta'\omega\hat{\phi}$

Distance between $\vec{r}'$ and the z-axis $= R\sin\theta'$ ← (Direction is $\hat{\rho}$ of Cylindrical Coordinates.)

$$\vec{L} = \int_{\theta'=0}^{\pi}\int_{\phi'=0}^{2\pi}\left(\frac{M}{4\pi R}\right)\sin\theta'\omega(R\sin\theta')\hat{z}\;R^2\sin\theta' d\theta' d\phi' \quad \leftarrow (\hat{z} = \hat{\rho}\times\hat{\phi})$$

$$= \frac{MR^2\omega}{4\pi}\hat{z}\int_{\theta'=0}^{\pi}\sin^3\theta' d\theta'\int_{\phi'=0}^{2\pi}d\phi' \Rightarrow \vec{L} = \frac{2}{3}MR^2\omega\hat{z}$$

h) $\vec{m} = \frac{1}{3}QR^2\omega\hat{z} = \frac{1}{2}\left(\frac{Q}{M}\right)\vec{L}$

*Digression: For an electron, $e = -1.6\times10^{-19}$ C and $M = 9.11\times10^{-31}$ Kg

$\Rightarrow \vec{m} = -0.88\times10^{11}\vec{L} = -0.927\times10^{-23}(\vec{L}/\hbar)$ ← ($\hbar = \frac{h}{2\pi}$ is planck's Constant)

The Constant $\mu_B = 0.927\times10^{-23}$ Amp·m^2 is known as a Bohr magneton, the quantum of magnetic moment. In quantum mechanics, it turns out that the correct expression for $\vec{m}$ of an electron due to its spin angular momentum is nearly twice the value given by the above equation. In other words, the spin angular momentum of an electron is $\frac{1}{2}\hbar$, whereas its magnetic dipole moment is $\sim\mu_B$.