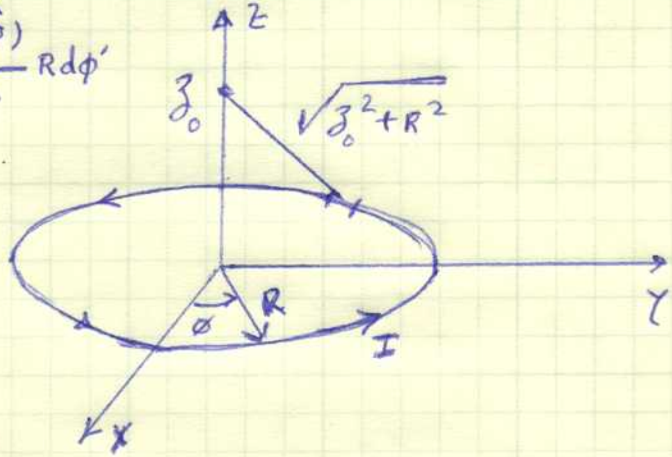
$$\vec{H}(0,0,z_0)=\frac{1}{4\pi}\int_{\phi'=0}^{2\pi}\frac{I\hat{\phi}\times(z_0\hat{z}-R\hat{\rho})}{(z_0^2+R^2)^{3/2}}R\,d\phi'$$



$$=\frac{RI}{4\pi(z_0^2+R^2)^{3/2}}\int_{\phi'=0}^{2\pi}(z_0\hat{\rho}+R\hat{z})\,d\phi'$$

$$=\frac{R^2I}{2(z_0^2+R^2)^{3/2}}\hat{z}.$$

In this derivation we have used cylindrical coordinates to express $\vec{J}(\vec{r}\,')$, $\vec{r}$, and $\vec{r}\,'$ as follows:

$$\vec{J}(\vec{r}\,')\,dv'=\vec{J}(\vec{r}\,')\,ds'\,d\ell'=(I\hat{\phi})\,d\ell'=(I\hat{\phi})(R\,d\phi')=RI\hat{\phi}\,d\phi'$$

$$\vec{r}=z_0\hat{z}$$

$$\vec{r}\,'=R\hat{\rho}$$

$$|\vec{r}-\vec{r}\,'|=|z_0\hat{z}-R\hat{\rho}|=\sqrt{z_0^2+R^2}$$

We have also used the fact that $\hat{\phi}\times\hat{z}=\hat{\rho}$, $\hat{\phi}\times\hat{\rho}=-\hat{z}$,

and $\int_{\phi'=0}^{2\pi}\hat{\rho}\,d\phi'=0$.