Problem 20) $\vec{B} = \vec{\nabla} \times \vec{A} = \vec{\nabla} \times \left(\frac{\mu_0}{4\pi} \int_{V'} \frac{\vec{J}(\vec{r}')}{|\vec{r}-\vec{r}'|} dv' \right) = \frac{\mu_0}{4\pi} \int_{V'} \vec{\nabla} \times \left[\frac{\vec{J}(\vec{r}')}{|\vec{r}-\vec{r}'|} \right] dv'$

$$= \frac{\mu_0}{4\pi} \int_{V'} \left\{ \frac{1}{|\vec{r}-\vec{r}'|} \vec{\nabla} \times \vec{J}(\vec{r}') + \left[\vec{\nabla} \left(\frac{1}{|\vec{r}-\vec{r}'|} \right) \right] \times \vec{J}(\vec{r}') \right\} dv'$$

Note that the $\vec{\nabla}$ operator operates on the variable $\vec{r}$ <u>not</u> $\vec{r}'$!

Therefore, $\vec{\nabla} \times \vec{J}(\vec{r}') = 0$, because $\vec{J}(\vec{r}')$ is <u>not</u> a function of $\vec{r}$.

From the preceding problem (Problem 3-8) we know that

$\vec{\nabla}\left(\frac{1}{|\vec{r}|} \right) = -\frac{\vec{r}}{|\vec{r}|^3}$. A change of variable to $(\vec{r}-\vec{r}')$ shows

that $\vec{\nabla}\left(\frac{1}{|\vec{r}-\vec{r}'|} \right) = -\frac{(\vec{r}-\vec{r}')}{|\vec{r}-\vec{r}'|^3}$. We thus have:

$$\vec{B} = -\frac{\mu_0}{4\pi} \int_{V'} \frac{\vec{r}-\vec{r}'}{|\vec{r}-\vec{r}'|^3} \times \vec{J}(\vec{r}') \, dv' = \frac{\mu_0}{4\pi} \int_{V'} \frac{\vec{J}(\vec{r}') \times (\vec{r}-\vec{r}')}{|\vec{r}-\vec{r}'|^3} dv'$$