Solutions **Opti 501**

Problem 19)

a) The only leg of the rectangle that contributes to the $\oint \vec{H}\cdot d\vec{\ell}$ is the vertical leg inside the coil. (Outside the coil $\vec{H}=0$, and on the horizontal legs $\vec{H}$ is $\perp$; also if the coil is long enough, the contributions of the two horizontal legs would cancel each other out.) Thus $\oint \vec{H}\cdot d\vec{\ell} = H\Delta z$, where H is the magnitude of the field inside the coil, and Δz is the length of the vertical leg of the rectangle. The current crossing the rectangle is $I(N/\ell)\Delta z$, which must be equal to $\oint \vec{H}\cdot d\vec{\ell}$. Therefore, $H = NI/\ell$.

b) Magnetic energy = (energy density) · (Volume of the coil)

$= (\frac{1}{2}\mu_0 H^2)\cdot(a\ell) = \frac{1}{2}\mu_0 (NI/\ell)^2 (a\ell) = \frac{1}{2}\mu_0 N^2 (a/\ell) I^2$

$\Rightarrow$ Total magnetic energy contained in the coil $= \frac{1}{2} L I^2$

c) Magnetic flux within <u>each</u> turn of the coil $= Ba = \mu_0 Ha$.

Voltage induced around each turn of the coil $= \frac{\partial}{\partial t}(Ba)$

$= \mu_0 N (a/\ell) \frac{dI(t)}{dt}$.

Voltage induced in all N turns $= N\frac{\partial}{\partial t}(Ba) = \mu_0 N^2 (a/\ell)\frac{dI(t)}{dt}$

$\Rightarrow$ Induced voltage between solenoid terminals $= L\frac{dI(t)}{dt}$