

Problem 16) The symmetry of the problem dictates that the vector potential $\mathbf{A}(\mathbf{r})$ be confined to the ρz -plane in cylindrical coordinates, that is, $A_\phi(\mathbf{r})=0$. Moreover, $\mathbf{A}(\mathbf{r})$ cannot depend on the azimuthal coordinate ϕ . Consequently, $\mathbf{A}(\mathbf{r})=A_\rho(\rho, z)\hat{\boldsymbol{\rho}}+A_z(\rho, z)\hat{\mathbf{z}}$. Since $\mathbf{B}=\mu_0\mathbf{H}=\nabla\times\mathbf{A}$, the magnetic field is readily seen to have only an azimuthal component. Thus, when invoking Ampere's law, the symmetry of the toroidal coil allows one to consider only circular loops centered on the z -axis.

If the loop radius is less than R_1 or greater than R_2 , the total current crossing the loop will be zero; therefore, $H_\phi=0$ in these regions.

If the loop of radius ρ is inside the coil, we will have $\oint\mathbf{H}\cdot d\boldsymbol{\ell}=2\pi\rho H_\phi=NI$, which yields $H_\phi(\rho)=NI/(2\pi\rho)$; $R_1<\rho<R_2$. Note that, unlike cylindrical solenoids, the azimuthal field of a toroidal coil is *not* uniform but varies inversely with ρ , the distance from the toroid's central axis.
