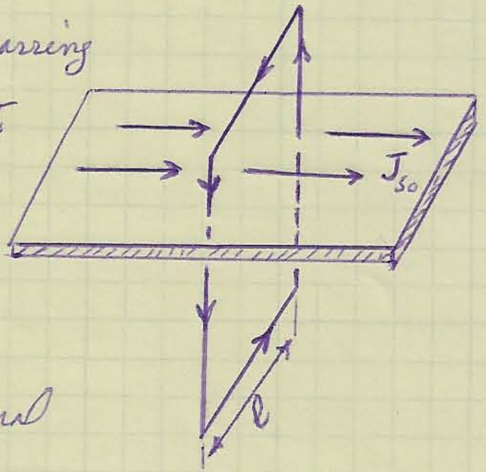
a) Consider a rectangular loop perpendicular to the current-carrying sheet and also perpendicular to the direction of current density $\vec{J}_s$. From symmetry, $\vec{H}$ must be parallel to the current-carrying sheet and perpendicular to $\vec{J}_s$. Using the integral



form of Ampère's law, $\oint \vec{H} \cdot d\vec{\ell} = I$ over the rectangular loop, we find the contributions of the vertical legs to be zero, while the horizontal legs at the top and bottom contribute equally to the loop integral; therefore,

$H_x(x,y,z>0)\,\ell - H_x(x,y,z<0)\,\ell = J_{so}\,\ell \Rightarrow \vec{H}(\vec{r}) = \pm\frac{1}{2} J_{so}\,\hat{x}$

Above the sheet — Below the sheet

+ sign when $\vec{r}$ is above the sheet
− sign when $\vec{r}$ is below the sheet

b) From symmetry $\vec{A}(\vec{r})$ cannot have any dependence on x or y. Moreover, it must be directed along the y-axis, because $\vec{J}$ everywhere is along $\hat{y}$. Thus $\vec{A}(\vec{r}) = A_y(z)\,\hat{y}$. Consequently:

$$\vec{\nabla}\times\vec{A} = \vec{B} = \mu_0\vec{H} \Rightarrow -\frac{\partial}{\partial z}A_y(z)\,\hat{x} = \mu_0 H_x(\vec{r})\,\hat{x} \Rightarrow A_y(z) = \begin{cases} -\frac{1}{2}\mu_0 J_{so} z & z>0 \\ +\frac{1}{2}\mu_0 J_{so} z & z<0 \end{cases}$$

In compact form: $\vec{A}(\vec{r}) = -\frac{1}{2}\mu_0 |z|\, J_{so}\,\hat{y}$ ← Note: $\vec{A}$ and $\vec{J}$ are in opposite directions because the integration constant has been ignored.

c) The magnetic field is the sum of the fields produced by the two current-carrying sheets. Therefore, $\vec{H}(\vec{r}) = \begin{cases} -J_{so}\,\hat{x} & \leftarrow \text{Between the sheets} \\ 0 & \leftarrow \text{outside} \end{cases}$

d) Change in the stored magnetic field energy $= \frac{1}{2}\mu_0 |\vec{H}|^2 a(d_1-d_0) = \frac{1}{2}\mu_0 J_{so}^2 a(d_1-d_0)$.

a: sheet area

Work done by the upper plate on the outside world $= (\frac{1}{2}\mu_0 J_{so})(J_{so})\,a(d_1-d_0)$

$\frac{1}{2}\mu_0 J_{so}$: $\vec{B}$-field of lower plate on the upper plate; J_{so}: current density of upper plate

The total energy provided by the batteries must be the sum of the above energies, namely, $\mu_0 J_{so}^2 a(d_1-d_0)$.

The $\vec{E}$-field acting on the electrons in the upper sheet, when the sheet moves up at a velocity $v(t)$, is $\vec{v}(t)\times\vec{B} = -\frac{1}{2}\mu_0 J_{so} v(t)\,\hat{y}$. This field, when integrated along the y-axis, yields the required voltage $V_1(t)$ to maintain the current in the upper sheet. The energy supplied by the upper battery is thus $\int_0^T V_1(t) I(t)\,dt = \frac{1}{2}\mu_0 J_{so}^2 a\int_0^T v(t)\,dt = \frac{1}{2}\mu_0 J_{so}^2 a(d_1-d_0)$. Here T is the time it takes for the distance between the plates to go from d_0 to d_1. As for the lower plate, it does not move and the $\vec{B}$-field acting on it does not change either. However, $\vec{E} = -\partial\vec{A}/\partial t$, generated by the movement of the upper plate, causes an identical voltage, $V_2(t) = V_1(t)$, in the lower plate. The energy supplied by $V_2(t)$ is thus $\frac{1}{2}\mu_0 J_{so}^2 a(d_1-d_0)$.