14) a) This is both an electrostatic problem (because σ_s is not a function of time) and a magneto-static problem (because J_s is not a function of time, and no charge is accumulating anywhere).

Since the charge density distribution σ_s is the same as that in Problem 1, the scalar potential ψ is the same. Moreover, $\vec{E} = -\vec{\nabla}\psi - \partial\vec{A}/\partial t$, but in magneto-statics $\vec{A}$ is not a function of time, hence $\vec{E} = -\vec{\nabla}\psi$ and, therefore, $\vec{E}$ is also the same as in Problem 1.

b) The linear velocity of the disk at a radius ρ is $\vec{V} = \rho\omega\hat{\phi}$.

Therefore, $\vec{J}_s(\rho) = \sigma_s\vec{V} = \sigma_s\rho\omega\hat{\phi}$, $\quad (\rho \leq R)$.

c) In cylindrical coordinates $\vec{\nabla}\cdot\vec{J} = \frac{1}{\rho}\frac{\partial}{\partial\rho}(\rho \overset{0}{\cancel{J_\rho}}) + \frac{1}{\rho}\frac{\partial}{\partial\phi}J_\phi + \frac{\partial}{\partial z}\overset{0}{\cancel{J_z}} =$

$\frac{1}{\rho}\frac{\partial}{\partial\phi}J_\phi = \frac{1}{\rho}\frac{\partial}{\partial\phi}(\sigma_s\rho\omega) = 0.$

Also, σ_s is time-independent; therefore $\frac{\partial\sigma_s}{\partial t} = 0. \Rightarrow \vec{\nabla}\cdot\vec{J}_s + \frac{\partial\sigma_s}{\partial t} = 0$ ✓

d) First, $\vec{A}(\rho,\phi,z)$ does not depend on ϕ. Second, $\vec{A}(\rho,z)$ does not have a z-component, because $\vec{J}_s$ does not have a z-component. Third, $\vec{A}$ does not have a ρ-component, because for each point on the disk that contributes an A_ρ at an observation point, there is always another point on the disk which cancels that contribution to A_ρ.

We conclude that the vector potential field is $A_\phi(\rho,z)\hat{\phi}$.

Moreover, on the z-axis the vector potential is zero, i.e., $A_\phi(0,z) = 0$.

Finally, there is symmetry with respect to the xy-plane, namely,

$A_\phi(\rho,z) = A_\phi(\rho,-z)$.

e) $\vec{B} = \vec{\nabla}\times\vec{A} \Rightarrow \mu_0 \vec{H} = \left(\frac{1}{\rho}\cancelto{0}{\frac{\partial A_z}{\partial\phi}} - \frac{\partial A_\phi}{\partial z}\right)\hat{\rho} + \left(\cancelto{0}{\frac{\partial A_\rho}{\partial z}} - \cancelto{0}{\frac{\partial A_z}{\partial\rho}}\right)\hat{\phi} + \frac{1}{\rho}\left(\frac{\partial}{\partial\rho}(\rho A_\phi) - \cancelto{0}{\frac{\partial A_\rho}{\partial\phi}}\right)\hat{z}$

$\Rightarrow \mu_0 \vec{H} = -\frac{\partial A_\phi}{\partial z}\hat{\rho} + \left(\frac{A_\phi}{\rho} + \frac{\partial A_\phi}{\partial\rho}\right)\hat{z}.$

✓ Thus $H_\phi = 0$, $\mu_0 H_\rho(\rho,z) = -\frac{\partial A_\phi}{\partial z}$, and $\mu_0 H_z(\rho,z) = \frac{A_\phi(\rho,z)}{\rho} + \frac{\partial}{\partial\rho}A_\phi(\rho,z)$.

✓ On the z-axis, $H_\rho = 0$ because $A_\phi(0,z) = 0$.

✓ $H_\rho(\rho,z) = -H_\rho(\rho,-z)$ because of the symmetry of A_ϕ with respect to the xy-plane.

✓ $H_z(\rho,z) = H_z(\rho,-z)$, again because of the symmetry of A_ϕ w.r.t. xy-plane.

f) First, $H_\phi = 0$ everywhere. Second, $H_\rho(\rho,z) = -H_\rho(\rho,-z)$ implies that at $z=0$ we must have $H_\rho(\rho,0) = -H_\rho(\rho,0)$, which is possible only if $H_\rho(\rho,0) = 0$. Therefore, at $(\rho > R, \phi, z=0)$ we have $H_\phi = H_\rho = 0$.

g) Applying Ampere's law, $\vec{\nabla}\times\vec{H} = \vec{J} + \frac{\partial\vec{D}}{\partial t}$, to a small rectangular path that crosses the disk, we find that $H_\rho^{(A)} - H_\rho^{(B)} = J_s = \sigma_s \rho\omega \Rightarrow$

$H_\rho^{(A)} = -H_\rho^{(B)} = \frac{1}{2}\sigma_s\rho\omega.$

h) We can only say that $H_z^{(A)} = H_z^{(B)}$ from the symmetry discussed in part (e), or from the boundary condition derived from $\vec{\nabla}\cdot\vec{B} = 0$ that asserts that the perpendicular component of $\vec{B}$ is always continuous.

i) For a ring of radius ρ and width $\Delta\rho$, the current I is $J_s\Delta\rho = \sigma_s\rho\omega\Delta\rho$, and the loop area is $\pi\rho^2$. Therefore, $\Delta\vec{m} = (\pi\rho^2)(\sigma_s\rho\omega)\Delta\rho\,\hat{z}$.

Consequently $\vec{m} = \int_{\rho=0}^{R} \pi\sigma_s\omega\rho^3\hat{z}\,d\rho = \frac{1}{4}\pi\sigma_s\omega R^4\hat{z}$. This definition of $\vec{m}$ is consistent with $\vec{B} = \mu_0(\vec{H} + \vec{m})$. It must be multiplied by μ_0 if $\vec{B} = \mu_0\vec{H} + \vec{m}$.