13) a) There is no dependence on ϕ, thus ψ is a function of ρ and θ only. Moreover, there is symmetry between points above and below the XY-plane, having the same x and y coordinates, but differing signs for the z-coordinate, namely, $\pm z$. Thus $\psi(\rho,\theta) = \psi(\rho,\pi-\theta)$.

b) $\vec{E} = -\vec{\nabla}\psi \Rightarrow \vec{E}(\rho,\theta,\phi) = -\frac{\partial\psi}{\partial\rho}\hat{\rho} - \frac{1}{\rho}\frac{\partial\psi}{\partial\theta}\hat{\theta} - \frac{1}{\rho\sin\theta}\frac{\partial\psi}{\partial\phi}\hat{\phi}$.

Since ψ is not dependent on ϕ, we conclude that $E_\phi = 0$. Thus, $E_\rho(\rho,\theta) = -\frac{\partial}{\partial\rho}\psi(\rho,\theta)$ and $E_\theta(\rho,\theta) = -\frac{1}{\rho}\frac{\partial}{\partial\theta}\psi(\rho,\theta)$. Symmetry with respect to the xy-plane shows that $E_\rho(\rho,\theta) = E_\rho(\rho,\pi-\theta)$ and $E_\theta(\rho,\theta) = -E_\theta(\rho,\pi-\theta)$.

c) In general, we have already seen that $E_\phi = 0$ everywhere. As for E_θ, the odd symmetry obtained in part (b), namely, $E_\theta(\rho,\theta) = -E_\theta(\rho,\pi-\theta)$, dictates that at $\theta = \pi/2$ we must have $E_\theta(\rho,\pi/2) = -E_\theta(\rho,\pi/2)$. This is possible only if $E_\theta(\rho,\pi/2) = 0$. We conclude, therefore, that at $(\rho>R, \theta=\pi/2, \phi)$ $E_\theta = E_\phi = 0$.

d) Using Gauss' law, $\vec{\nabla}\cdot\vec{E} = \rho/\varepsilon_0$, applied to a small pillbox at the surface of the disk, and the symmetry condition that $E_\theta^{\text{above}} = -E_\theta^{\text{below}}$ we find $E_\theta^{(A)} = -E_\theta^{(B)} = -\frac{1}{2}\sigma_s/\varepsilon_0$.

e) We can only say that $E_\rho^{(A)} = E_\rho^{(B)}$, either from symmetry discussed in part (b), or from the boundary condition derived from $\vec{\nabla}\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}$ that asserts that the tangential component of $\vec{E}$ is always continuous.

Note that the discontinuity of $E_\theta(\rho<R, \theta=\frac{\pi}{2}, \phi)$ at the disk surface does not imply a discontinuity for $\psi(\rho,\theta)$ at the disk surface, but only for its first derivative.