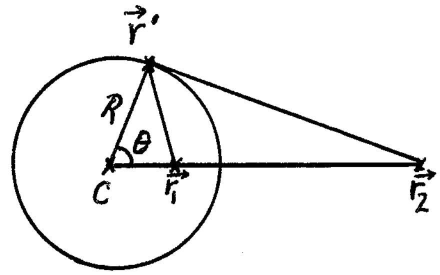
Problem 10)

a) $|\vec{r}_1-\vec{r}\,'|^2=(\vec{r}_1-\vec{r}\,')\cdot(\vec{r}_1-\vec{r}\,')$

$= r_1^2+R^2-2Rr_1\cos\theta$



Similarly, $|\vec{r}_2-\vec{r}\,'|^2=r_2^2+R^2-2Rr_2\cos\theta$.

$$\frac{|\vec{r}_1-\vec{r}\,'|^2}{|\vec{r}_2-\vec{r}\,'|^2}=\frac{r_1^2+R^2-2Rr_1\cos\theta}{r_2^2+R^2-2Rr_2\cos\theta}=\alpha$$ ← We must find conditions that make α independent of θ.

$\Rightarrow r_1^2+R^2-2Rr_1\cos\theta=\alpha r_2^2+\alpha R^2-2\alpha Rr_2\cos\theta \Rightarrow 2R\cos\theta(\alpha r_2-r_1)=R^2(\alpha-1)+\alpha r_2^2-r_1^2$.

For θ to be irrelevant, the two sides of the preceding equation must be zero. Therefore,

$$\begin{cases}\alpha r_2-r_1=0 \Rightarrow \alpha=r_1/r_2\\ R^2(\alpha-1)+\alpha r_2^2-r_1^2=0 \Rightarrow R^2(\frac{r_1}{r_2}-1)+r_1r_2-r_1^2=0 \Rightarrow R^2(\frac{r_1}{r_2}-1)+r_1r_2(1-\frac{r_1}{r_2})=0 \Rightarrow\end{cases}$$

$(R^2-r_1r_2)(\frac{r_1}{r_2}-1)=0 \Rightarrow r_1r_2=R^2$ ← This relation provides a unique $\vec{r}_2$ for each $\vec{r}_1$.

For each pair of points $(\vec{r}_1,\vec{r}_2)$, one inside the sphere, the other outside, such that $r_1r_2=R^2$, we'll have $\frac{|\vec{r}_1-\vec{r}\,'|}{|\vec{r}_2-\vec{r}\,'|}=\sqrt{\frac{r_1}{r_2}}=\frac{r_1}{R}$, irrespective of the value of θ; that is, irrespective of the location of $\vec{r}\,'$ on the surface of the sphere.

b) $$\psi(\vec{r}_1)=\frac{1}{4\pi\epsilon_0}\int_{\theta=0}^{\pi}\int_{\phi=0}^{2\pi}\frac{\sigma_s(R,\theta,\phi)}{|\vec{r}\,'-\vec{r}_1|}R^2\sin\theta\,d\theta\,d\phi$$ ←

$$\psi(\vec{r}_2)=\frac{1}{4\pi\epsilon_0}\int_{\theta=0}^{\pi}\int_{\phi=0}^{2\pi}\frac{\sigma_s(R,\theta,\phi)}{|\vec{r}\,'-\vec{r}_2|}R^2\sin\theta\,d\theta\,d\phi=(r_1/R)\,\psi(\vec{r}_1)$$