Problem 8) At the shell surface, $\psi(\boldsymbol{r})$ must be continuous. Therefore,

$$(A\cos\theta)/R^2 = BR\cos\theta \qquad \rightarrow \qquad B = A/R^3.$$

The boundary condition derived from Maxwell's first equation requires that, at the surface of the shell, the surface charge-density $\sigma(\boldsymbol{r})$ be equal to the discontinuity in $\varepsilon_o E_\perp$, where, given the geometry of the present problem, $E_\perp = E_r$. Therefore,

$$\sigma_0 \cos\theta = \varepsilon_o \left[E_r^{(\text{out})} - E_r^{(\text{in})}\right].$$

Since $E_r = -\partial\psi/\partial r$ evaluated at $\boldsymbol{r} = (R, \theta, \phi)$, we will have

$$\sigma_0 \cos\theta = \varepsilon_o \left[-\frac{\partial}{\partial r}\left(\frac{A\cos\theta}{r^2}\right)\Big|_{r=R} + \frac{\partial}{\partial r}(Br\cos\theta)|_{r=R}\right]$$

$$\rightarrow \quad \sigma_0 = \varepsilon_o\,[(2A/R^3) + B] = 3\varepsilon_o A/R^3 \quad \rightarrow \quad A = \sigma_0 R^3/(3\varepsilon_o).$$

The scalar potential *outside* the sphere is thus $\psi(\boldsymbol{r}) = \sigma_0 R^3 \cos\theta/(3\varepsilon_o r^2)$. Compared with the scalar potential derived in Chapter 4, Example 8 for an electric point-dipole $\boldsymbol{p} = p_0\hat{\boldsymbol{z}}$, we observe that the charged spherical shell in the present problem is equivalent to a dipole $\boldsymbol{p} = (4\pi R^3\sigma_0/3)\hat{\boldsymbol{z}}$, located at the origin of the coordinate system.

Inside the shell, $\psi(\boldsymbol{r}) = Br\cos\theta = (\sigma_0/3\varepsilon_o)(r\cos\theta) = (\sigma_0/3\varepsilon_o)z$, which is a linear function of the vertical coordinate z. Therefore, the E-field inside the shell is given by

$$\boldsymbol{E}(\boldsymbol{r}) = -\boldsymbol{\nabla}\psi(\boldsymbol{r}) = -(\partial\psi/\partial z)\hat{\boldsymbol{z}} = -(\sigma_0/3\varepsilon_o)\hat{\boldsymbol{z}}.$$

The internal field of the spherical dipole is thus uniform, having a magnitude of $\sigma_0/3\varepsilon_o$ and pointing along the negative z-axis.