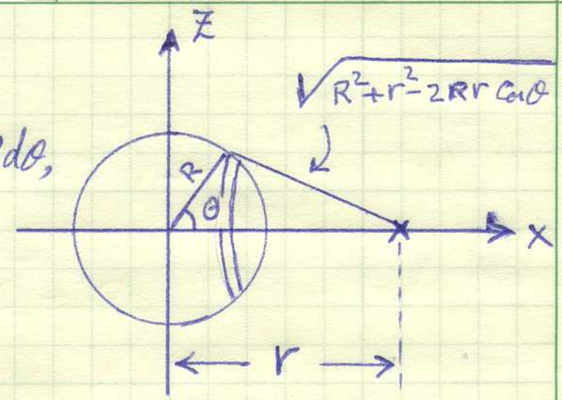
Problem 7)

The ring of charge shown has radius $R\sin\theta$, area $(2\pi R\sin\theta)R\,d\theta$, and charge $2\pi R^2\sigma\sin\theta\,d\theta$. The potential function is thus given by



$$\psi(r) = \frac{1}{4\pi\epsilon_0}\int_{\theta=0}^{\pi} \frac{2\pi R^2\sigma\sin\theta\,d\theta}{\sqrt{R^2+r^2-2Rr\cos\theta}}$$

$$= \frac{2\pi R^2\sigma}{4\pi\epsilon_0}\left(\frac{1}{Rr}\sqrt{R^2+r^2-2Rr\cos\theta}\right)\Bigg|_{\theta=0}^{\pi} = \frac{2\pi R^2\sigma}{4\pi\epsilon_0}\,\frac{\sqrt{(R+r)^2}-\sqrt{(R-r)^2}}{Rr}$$

The total charge of the spherical shell is $Q = 4\pi R^2\sigma$. Therefore,

square root is always positive

$$\psi(r) = \frac{\frac{1}{2}Q}{4\pi\epsilon_0}\,\frac{R+r-|R-r|}{Rr}$$

$$= \begin{cases} \dfrac{Q}{4\pi\epsilon_0 R} & r \le R \\[2ex] \dfrac{Q}{4\pi\epsilon_0 r} & r \ge R \end{cases}$$

Thus inside the shell $\psi(r)$ is constant; outside the shell it drops with $1/r$, same as a point charge Q located at the origin of coordinates. Note that $\psi(r)$ is continuous at the shell surface. The E-field is the minus gradient of $\psi(r)$, namely $\vec{E}(r) = -\vec{\nabla}\psi(r)$. Inside the shell, the E-field is zero. outside the shell it is $\vec{E}(r) = -\frac{\partial\psi}{\partial r}\hat{r} = \frac{Q}{4\pi\epsilon_0 r^2}\hat{r}$.