Problem 6) a) $\boldsymbol{E}(x, y, z=0) = -\frac{2Q}{4\pi\varepsilon_0}\left(\frac{\cos\theta}{r^2}\right)\hat{\boldsymbol{z}} = -\frac{2Qd}{4\pi\varepsilon_0 r^3}\hat{\boldsymbol{z}} = -\frac{Qd}{2\pi\varepsilon_0(x^2+y^2+d^2)^{3/2}}\hat{\boldsymbol{z}}.$

b) $\psi(x, y, z=0) = \frac{Q}{4\pi\varepsilon_0 r} - \frac{Q}{4\pi\varepsilon_0 r} = 0.$

Alternatively, since the E-field is perpendicular to the xy-plane, the integral of $\boldsymbol{E}\cdot \mathrm{d}\boldsymbol{\ell}$ from any point (x_0, y_0) in the xy-plane to infinity, taken along any arbitrary path in the xy-plane, will be zero. By definition, this integral is the scalar potential at (x_0, y_0); therefore, $\psi(x, y, z=0) = 0$.

c) This is the so-called "method of images." The E-field in the upper half-space $z \geq 0$ is the same for the systems depicted in figures (a) and (b). The surface charge-density is proportional to the perpendicular component of the E-field at the surface of the conductor. We thus have

$$\sigma(x, y) = \varepsilon_0 E_\perp(x, y, z=0) = -\frac{Qd}{2\pi(x^2+y^2+d^2)^{3/2}}.$$

Note: The integral of $\sigma(x, y)$ over the entire xy-plane equals $-Q$.