Problem 4.45) a) Transforming the equations to the Fourier domain, we find

$$\mathrm{i}\varepsilon_o \boldsymbol{k}\cdot\boldsymbol{E}(\boldsymbol{k},\omega) = \rho_{\text{free}}(\boldsymbol{k},\omega) - \mathrm{i}\boldsymbol{k}\cdot\boldsymbol{P}(\boldsymbol{k},\omega), \tag{1a}$$

$$\mathrm{i}\boldsymbol{k}\times\boldsymbol{H}(\boldsymbol{k},\omega) = \boldsymbol{J}_{\text{free}}(\boldsymbol{k},\omega) - \mathrm{i}\omega\boldsymbol{P}(\boldsymbol{k},\omega) - \mathrm{i}\varepsilon_o\omega\boldsymbol{E}(\boldsymbol{k},\omega), \tag{1b}$$

$$\mathrm{i}\boldsymbol{k}\times\boldsymbol{E}(\boldsymbol{k},\omega) = \mathrm{i}\omega\boldsymbol{M}(\boldsymbol{k},\omega) + \mathrm{i}\mu_o\omega\boldsymbol{H}(\boldsymbol{k},\omega), \tag{1c}$$

$$\mathrm{i}\mu_o\boldsymbol{k}\cdot\boldsymbol{H}(\boldsymbol{k},\omega) = -\mathrm{i}\boldsymbol{k}\cdot\boldsymbol{M}(\boldsymbol{k},\omega). \tag{1d}$$

b) Cross-multiplying $\boldsymbol{k}$ into Eqs.(1b) and (1c) yields

$$\boldsymbol{k}\times[\boldsymbol{k}\times\boldsymbol{H}(\boldsymbol{k},\omega)] = -\mathrm{i}\boldsymbol{k}\times\boldsymbol{J}_{\text{free}}(\boldsymbol{k},\omega) - \omega\boldsymbol{k}\times\boldsymbol{P}(\boldsymbol{k},\omega) - \varepsilon_o\omega\boldsymbol{k}\times\boldsymbol{E}(\boldsymbol{k},\omega), \tag{2a}$$

$$\boldsymbol{k}\times[\boldsymbol{k}\times\boldsymbol{E}(\boldsymbol{k},\omega)] = \omega\boldsymbol{k}\times\boldsymbol{M}(\boldsymbol{k},\omega) + \mu_o\omega\boldsymbol{k}\times\boldsymbol{H}(\boldsymbol{k},\omega). \tag{2b}$$

The above equations may now be simplified if $\boldsymbol{k}\times\boldsymbol{E}$ from Eq.(1c) and $\boldsymbol{k}\times\boldsymbol{H}$ from Eq.(1b) are substituted into Eqs.(2a) and Eq.(2b), respectively, and also if the vector identity $\boldsymbol{A}\times(\boldsymbol{B}\times\boldsymbol{C}) = (\boldsymbol{A}\cdot\boldsymbol{C})\boldsymbol{B} - (\boldsymbol{A}\cdot\boldsymbol{B})\boldsymbol{C}$ is used to simplify the left-hand sides of Eqs.(2a) and (2b), as follows:

$$\begin{aligned}[\boldsymbol{k}\cdot\boldsymbol{H}(\boldsymbol{k},\omega)]\boldsymbol{k} - k^2\boldsymbol{H}(\boldsymbol{k},\omega) = -\mathrm{i}\boldsymbol{k}\times\boldsymbol{J}_{\text{free}}(\boldsymbol{k},\omega) - \omega\boldsymbol{k}\times\boldsymbol{P}(\boldsymbol{k},\omega)\\ -\varepsilon_o\omega^2[\boldsymbol{M}(\boldsymbol{k},\omega) + \mu_o\boldsymbol{H}(\boldsymbol{k},\omega)],\end{aligned} \tag{3a}$$

$$\begin{aligned}[\boldsymbol{k}\cdot\boldsymbol{E}(\boldsymbol{k},\omega)]\boldsymbol{k} - k^2\boldsymbol{E}(\boldsymbol{k},\omega) = \omega\boldsymbol{k}\times\boldsymbol{M}(\boldsymbol{k},\omega) - \mathrm{i}\mu_o\omega\boldsymbol{J}_{\text{free}}(\boldsymbol{k},\omega)\\ -\mu_o\omega^2[\boldsymbol{P}(\boldsymbol{k},\omega) + \varepsilon_o\boldsymbol{E}(\boldsymbol{k},\omega)].\end{aligned} \tag{3b}$$

Next, we substitute $\boldsymbol{k}\cdot\boldsymbol{H}$ from Eq.(1d) into Eq.(3a), and $\boldsymbol{k}\cdot\boldsymbol{E}$ from Eq.(1a) into Eq.(3b) to obtain

$$\begin{aligned}[(\omega/c)^2 - k^2]\boldsymbol{H}(\boldsymbol{k},\omega) = -\mathrm{i}\boldsymbol{k}\times\boldsymbol{J}_{\text{free}}(\boldsymbol{k},\omega) - \omega\boldsymbol{k}\times\boldsymbol{P}(\boldsymbol{k},\omega)\\ -\varepsilon_o\omega^2\boldsymbol{M}(\boldsymbol{k},\omega) + \mu_o^{-1}[\boldsymbol{k}\cdot\boldsymbol{M}(\boldsymbol{k},\omega)]\boldsymbol{k},\end{aligned} \tag{4a}$$

$$\begin{aligned}[(\omega/c)^2 - k^2]\boldsymbol{E}(\boldsymbol{k},\omega) = \mathrm{i}\varepsilon_o^{-1}\rho_{\text{free}}(\boldsymbol{k},\omega)\boldsymbol{k} - \mathrm{i}\mu_o\omega\boldsymbol{J}_{\text{free}}(\boldsymbol{k},\omega) + \omega\boldsymbol{k}\times\boldsymbol{M}(\boldsymbol{k},\omega)\\ -\mu_o\omega^2\boldsymbol{P}(\boldsymbol{k},\omega) + \varepsilon_o^{-1}[\boldsymbol{k}\cdot\boldsymbol{P}(\boldsymbol{k},\omega)]\boldsymbol{k}.\end{aligned} \tag{4b}$$

The final solutions for the electromagnetic fields $\boldsymbol{E}(\boldsymbol{k},\omega)$ and $\boldsymbol{H}(\boldsymbol{k},\omega)$ are thus given by

$$\boldsymbol{H}(\boldsymbol{k},\omega) = \frac{\mathrm{i}\boldsymbol{k}\times\boldsymbol{J}_{\text{free}}(\boldsymbol{k},\omega) + \omega\boldsymbol{k}\times\boldsymbol{P}(\boldsymbol{k},\omega) + \varepsilon_o\omega^2\boldsymbol{M}(\boldsymbol{k},\omega) - \mu_o^{-1}[\boldsymbol{k}\cdot\boldsymbol{M}(\boldsymbol{k},\omega)]\boldsymbol{k}}{k^2 - (\omega/c)^2}, \tag{5a}$$

$$\boldsymbol{E}(\boldsymbol{k},\omega) = \frac{-\mathrm{i}\varepsilon_o^{-1}\rho_{\text{free}}(\boldsymbol{k},\omega)\boldsymbol{k} + \mathrm{i}\mu_o\omega\boldsymbol{J}_{\text{free}}(\boldsymbol{k},\omega) + \mu_o\omega^2\boldsymbol{P}(\boldsymbol{k},\omega) - \varepsilon_o^{-1}[\boldsymbol{k}\cdot\boldsymbol{P}(\boldsymbol{k},\omega)]\boldsymbol{k} - \omega\boldsymbol{k}\times\boldsymbol{M}(\boldsymbol{k},\omega)}{k^2 - (\omega/c)^2}. \tag{5b}$$

It is not difficult to verify that the above expressions for E and H fields are the same as those obtained using the bound electric charge and current densities — with or without the introduction of scalar and vector potentials.