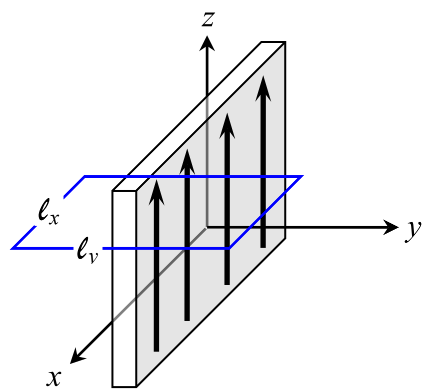

Problem 4.44) a) Because of symmetry, the H-field cannot depend on x or z. Take a rectangular loop $\ell_x \times \ell_y$ parallel to the xy-plane and write the integral form of Ampere's law, $\boldsymbol{\nabla} \times \boldsymbol{H} = \boldsymbol{J}_{\text{free}}$, for this loop. The contributions of ℓ_y to the loop integral cancel out, leaving only the contributions of ℓ_x on opposite sides of the current sheet. Therefore, $2H_x\ell_x = J_{\text{so}}\ell_x$, where $J_{\text{so}}\ell_x$ is the current crossing the loop. The magnitude of the H-field is thus independent of y, although its direction depends on whether y is positive or negative. Taking the right-hand rule into account, the final result is

$$\boldsymbol{H}(\boldsymbol{r},t) = -\tfrac{1}{2}\,\text{sign}(y)\,J_{so}\,\hat{\boldsymbol{x}}.$$

Note: Using symmetry and Maxwell's 4th equation, $\boldsymbol{\nabla}\cdot\boldsymbol{B} = 0$, it is easy to see why H_y must be zero everywhere: Take a cylinder whose axis is parallel to y and which the xz-plane cuts in the middle, then use the fact that the net flux of $\boldsymbol{H}$ into or out of the cylinder must be zero. Similarly, Maxwell's 2nd equation can be used to show that H_z is independent of y; the argument parallels that used above to evaluate H_x, except that the rectangular loop is now chosen in the yz-plane. Since H_z is already known to be independent of x and z, we conclude that it must be constant through the entire space. Showing that H_z is identically zero, however, requires the full solution of Maxwell's equations, which is done in part (b).

b) Fourier transforming the current density $\boldsymbol{J}(\boldsymbol{r},t) = J_{\text{so}}\delta(y)\,\hat{\boldsymbol{z}}$ yields

$$\boldsymbol{J}(\boldsymbol{k},\omega) = \int_{-\infty}^{\infty} J_{so}\delta(y)\,\hat{\boldsymbol{z}}\exp[-\mathrm{i}(\boldsymbol{k}\cdot\boldsymbol{r}-\omega t)]\,\mathrm{d}\boldsymbol{r}\,\mathrm{d}t = (2\pi)^3 J_{so}\,\delta(k_x)\,\delta(k_z)\,\delta(\omega)\,\hat{\boldsymbol{z}}.$$

The H-field is thus given by

$$\boldsymbol{H}(\boldsymbol{r},t) = \mu_o^{-1}\boldsymbol{B}(\boldsymbol{r},t) = \mu_o^{-1}\boldsymbol{\nabla}\times\boldsymbol{A}(\boldsymbol{r},t) = (2\pi)^{-4}\int_{-\infty}^{\infty}\frac{\mathrm{i}\boldsymbol{k}\times\boldsymbol{J}(\boldsymbol{k},\omega)}{k^2-(\omega/c)^2}\exp[\mathrm{i}(\boldsymbol{k}\cdot\boldsymbol{r}-\omega t)]\,\mathrm{d}\boldsymbol{k}\,\mathrm{d}\omega$$

$$= \frac{\mathrm{i}J_{so}}{2\pi}\int_{-\infty}^{\infty}\frac{(\boldsymbol{k}\times\hat{\boldsymbol{z}})\,\delta(k_x)\delta(k_z)\delta(\omega)}{k^2-(\omega/c)^2}\exp[\mathrm{i}(\boldsymbol{k}\cdot\boldsymbol{r}-\omega t)]\,\mathrm{d}\boldsymbol{k}\,\mathrm{d}\omega$$

$$= \frac{\mathrm{i}J_{so}}{2\pi}\int_{-\infty}^{\infty}\frac{k_y\hat{\boldsymbol{y}}\times\hat{\boldsymbol{z}}}{k_y^2}\exp(\mathrm{i}k_y y)\,\mathrm{d}k_y = \mathrm{i}J_{so}\hat{\boldsymbol{x}}\,(2\pi)^{-1}\int_{-\infty}^{\infty}\frac{\exp(\mathrm{i}k_y y)}{k_y}\,\mathrm{d}k_y = -\tfrac{1}{2}\,\text{sign}(y)J_{so}\hat{\boldsymbol{x}}.$$