Problem 4.41) a) For the incident plane-wave in the region $y \le 0$, we have

$$\boldsymbol{E}^{(\text{inc})}(\boldsymbol{r},t) = -\boldsymbol{\nabla}\psi - \partial\boldsymbol{A}/\partial t = \omega_0 A_0 \hat{\boldsymbol{z}}\cos(k_0 y - \omega_0 t),$$

$$\boldsymbol{H}^{(\text{inc})}(\boldsymbol{r},t) = \mu_0^{-1}\boldsymbol{B}(\boldsymbol{r},t) = \mu_0^{-1}\boldsymbol{\nabla}\times\boldsymbol{A}(\boldsymbol{r},t) = \mu_0^{-1}(\partial A_z/\partial y)\hat{\boldsymbol{x}}$$

$$= \mu_0^{-1}k_0 A_0 \hat{\boldsymbol{x}}\cos(k_0 y - \omega_0 t) = (\omega_0 A_0/Z_0)\hat{\boldsymbol{x}}\cos(k_0 y - \omega_0 t).$$

For the reflected plane-wave (again in the region $y \le 0$), we have

$$\boldsymbol{E}^{(\text{ref})}(\boldsymbol{r},t) = -\boldsymbol{\nabla}\psi - \partial\boldsymbol{A}/\partial t = -\omega_0 A_0 \hat{\boldsymbol{z}}\cos(k_0 y + \omega_0 t),$$

$$\boldsymbol{H}^{(\text{ref})}(\boldsymbol{r},t) = \mu_0^{-1}\boldsymbol{B}(\boldsymbol{r},t) = \mu_0^{-1}\boldsymbol{\nabla}\times\boldsymbol{A}(\boldsymbol{r},t) = \mu_0^{-1}(\partial A_z/\partial y)\hat{\boldsymbol{x}}$$

$$= \mu_0^{-1}k_0 A_0 \hat{\boldsymbol{x}}\cos(k_0 y + \omega_0 t) = (\omega_0 A_0/Z_0)\hat{\boldsymbol{x}}\cos(k_0 y + \omega_0 t).$$

b) In the plane $y = 0$ at the front facet of the mirror, the total E-field and the total H-field are given by

$$\boldsymbol{E}^{(\text{total})}(x, y = 0, z, t) = \boldsymbol{E}^{(\text{inc})} + \boldsymbol{E}^{(\text{ref})} = \omega_0 A_0 \hat{\boldsymbol{z}}\cos(-\omega_0 t) - \omega_0 A_0 \hat{\boldsymbol{z}}\cos(\omega_0 t) = 0,$$

$$\boldsymbol{H}^{(\text{total})}(x, y = 0, z, t) = \boldsymbol{H}^{(\text{inc})} + \boldsymbol{H}^{(\text{ref})} = 2(\omega_0 A_0/Z_0)\hat{\boldsymbol{x}}\cos(\omega_0 t).$$

There is no perpendicular E-field immediately before the mirror at $y = 0^-$. Also, inside the mirror, and specifically at $y = 0^+$, there are no E-fields. Maxwell's boundary condition relating the surface charge-density to the discontinuity of $\varepsilon_0\boldsymbol{E}_\perp$ at $y = 0$ thus yields $\sigma_s(x, z, t) = 0$.

The tangential H-field immediately before the mirror at $y = 0^-$ is $2(\omega_0 A_0/Z_0)\hat{\boldsymbol{x}}\cos(\omega_0 t)$. Since inside the mirror, and specifically at $y = 0^+$, there exist no H-fields, Maxwell's boundary condition relating the surface current-density $\boldsymbol{J}_s$ to the discontinuity of $\boldsymbol{H}_\parallel$ at $y = 0$ yields $\boldsymbol{J}_s(x, z, t) = 2(\omega_0 A_0/Z_0)\hat{\boldsymbol{z}}\cos(\omega_0 t)$. The amplitude of this surface current-density is thus given by $J_{s0} = 2(\omega_0 A_0/Z_0)$.

c) According to Example 10, Chapter 4, the $\boldsymbol{E}$ and $\boldsymbol{H}$ fields of the plane-wave propagating in the region $y \ge 0$ are $\boldsymbol{E}(\boldsymbol{r},t) = -\tfrac{1}{2}Z_0 J_{s0}\hat{\boldsymbol{z}}\cos(k_0 y - \omega_0 t)$ and $\boldsymbol{H}(\boldsymbol{r},t) = -\tfrac{1}{2}J_{s0}\hat{\boldsymbol{x}}\cos(k_0 y - \omega_0 t)$. We may also consider the vector potential of the field radiated into the shadow region, which is given by $\boldsymbol{A}(\boldsymbol{r},t) = -\tfrac{1}{2}(Z_0 J_{s0}/\omega_0)\hat{\boldsymbol{z}}\sin(k_0 y - \omega_0 t)$. Clearly, the field radiated into the shadow region is exactly cancelled out by the continuation beyond the PEC mirror of the incident beam.