4.35) First, we compute the bound current-density of the magnetic particle, as follows:

$$\boldsymbol{J}_{\text{bound}}^{(e)}(r,t) = \mu_0^{-1}\boldsymbol{\nabla}\times\boldsymbol{M}(\boldsymbol{r},t) = \mu_0^{-1}\boldsymbol{\nabla}\times\big[M_0\text{Sphere}(r/R)\underbrace{(\cos\theta\,\hat{\boldsymbol{r}} - \sin\theta\,\hat{\boldsymbol{\theta}})}_{\hat{\boldsymbol{z}}}\big]$$

$$= \mu_0^{-1}M_0\left\{-\frac{\partial[r\,\text{Sphere}(r/R)\sin\theta]}{r\partial r} - \frac{\partial[\text{Sphere}(r/R)\cos\theta]}{r\partial\theta}\right\}\hat{\boldsymbol{\phi}}$$

$$= \mu_0^{-1}M_0\left\{-\frac{[\text{Sphere}(r/R) - r\delta(r-R)]\sin\theta}{r} + \frac{\text{Sphere}(r/R)\sin\theta}{r}\right\}\hat{\boldsymbol{\phi}}$$

$$= \mu_0^{-1}M_0\delta(r-R)\sin\theta\,\hat{\boldsymbol{\phi}}. \qquad (1)$$

It is seen that the bound current is confined to the sphere's surface, flowing in the azimuthal direction $\hat{\boldsymbol{\phi}}$. The Fourier transform of the current-density is given by

$$\boldsymbol{J}_{\text{bound}}^{(e)}(\boldsymbol{k},\omega) = \int_{-\infty}^{\infty}\mu_0^{-1}M_0\delta(r-R)\sin\theta\,\hat{\boldsymbol{\phi}}\exp[-\mathrm{i}(\boldsymbol{k}\cdot\boldsymbol{r}-\omega t)]\,\mathrm{d}\boldsymbol{r}\mathrm{d}t$$

$$= \mu_0^{-1}M_0[2\pi\delta(\omega)]\int_{-\infty}^{\infty}\delta(r-R)(\hat{\boldsymbol{z}}\times\hat{\boldsymbol{r}})\exp(-\mathrm{i}\boldsymbol{k}\cdot\boldsymbol{r})\,\mathrm{d}\boldsymbol{r}$$

$$= 2\pi\mu_0^{-1}M_0\delta(\omega)\hat{\boldsymbol{z}}\times\int_{r=0}^{\infty}\int_{\theta=0}^{\pi}\delta(r-R)(\cos\theta\,\hat{\boldsymbol{k}})\exp(-\mathrm{i}kr\cos\theta)\,2\pi r^2\sin\theta\,\mathrm{d}\theta\mathrm{d}r$$

$$= 4\pi^2\mu_0^{-1}M_0\delta(\omega)(\hat{\boldsymbol{z}}\times\hat{\boldsymbol{k}})\int_{r=0}^{\infty}r^2\delta(r-R)\Big[\int_{\theta=0}^{\pi}\sin\theta\cos\theta\exp(-\mathrm{i}kr\cos\theta)\,\mathrm{d}\theta\Big]\mathrm{d}r$$

$$= -8\mathrm{i}\pi^2\mu_0^{-1}M_0\delta(\omega)(\hat{\boldsymbol{z}}\times\hat{\boldsymbol{k}})\int_{r=0}^{\infty}\frac{\sin(kr)-kr\cos(kr)}{k^2}\delta(r-R)\mathrm{d}r \quad \leftarrow \boxed{\text{G\&R } \mathbf{3.715}\text{-}11}$$

$$= -8\mathrm{i}\pi^2\mu_0^{-1}M_0\delta(\omega)(\hat{\boldsymbol{z}}\times\hat{\boldsymbol{k}})\,[\sin(kR)-kR\cos(kR)]/k^2. \qquad (2)$$

Now, in the Fourier domain, the vector potential is related to the total current-density as follows:

$$\boldsymbol{A}(\boldsymbol{k},\omega) = \mu_0\boldsymbol{J}_{\text{total}}^{(e)}(\boldsymbol{k},\omega)/[k^2-(\omega/c)^2]. \qquad (3)$$

An inverse Fourier transform thus yields the vector potential in the space-time domain, that is,

$$\boldsymbol{A}(\boldsymbol{r},t) = -\frac{8\mathrm{i}\pi^2M_0}{(2\pi)^4}\hat{\boldsymbol{z}}\times\int_{-\infty}^{\infty}\delta(\omega)\hat{\boldsymbol{k}}\left\{\frac{\sin(kR)-kR\cos(kR)}{k^2[k^2-(\omega/c)^2]}\right\}\exp[\mathrm{i}(\boldsymbol{k}\cdot\boldsymbol{r}-\omega t)]\,\mathrm{d}\boldsymbol{k}\mathrm{d}\omega$$

$$= -\frac{\mathrm{i}M_0}{2\pi^2}\hat{\boldsymbol{z}}\times\int_{-\infty}^{\infty}\hat{\boldsymbol{k}}\left[\frac{\sin(kR)-kR\cos(kR)}{k^4}\right]\exp(\mathrm{i}\boldsymbol{k}\cdot\boldsymbol{r})\,\mathrm{d}\boldsymbol{k}$$

$$= -\frac{\mathrm{i}M_0}{2\pi^2}(\hat{\boldsymbol{z}}\times\hat{\boldsymbol{r}})\int_{k=0}^{\infty}\int_{\theta=0}^{\pi}\cos\theta\left[\frac{\sin(kR)-kR\cos(kR)}{k^4}\right]\exp(\mathrm{i}kr\cos\theta)\,2\pi k^2\sin\theta\,\mathrm{d}\theta\mathrm{d}k$$

$$= -\mathrm{i}(M_0/\pi)(\hat{\boldsymbol{z}}\times\hat{\boldsymbol{r}})\int_{k=0}^{\infty}\left[\frac{\sin(kR)-kR\cos(kR)}{k^2}\right]\int_{\theta=0}^{\pi}\sin\theta\cos\theta\exp(\mathrm{i}kr\cos\theta)\,\mathrm{d}\theta\mathrm{d}k$$

$$= (2M_0/\pi r^2)(\hat{\boldsymbol{z}}\times\hat{\boldsymbol{r}})\int_0^{\infty}\frac{\sin(kR)-kR\cos(kR)}{k^2}\times\frac{\sin(kr)-kr\cos(kr)}{k^2}\mathrm{d}k \quad \leftarrow \boxed{\text{G\&R } \mathbf{3.715}\text{-}11}$$

$$= (2M_0/\pi r^2)(\hat{\boldsymbol{z}} \times \hat{\boldsymbol{r}}) \int_0^\infty \frac{\mathrm{d}}{\mathrm{d}k}[\sin(kR)/k] \times \frac{\mathrm{d}}{\mathrm{d}k}[\sin(kr)/k]\mathrm{d}k. \tag{4}$$

To evaluate the integral appearing in the above equation, note that $\sin(\alpha k)/k$ is the Fourier transform of ½$\mathrm{Rect}(x/2\alpha)$, which implies that $\mathrm{d}[\sin(\alpha k)/k]/\mathrm{d}k$ is the Fourier transform of $-(\mathrm{i}x/2)\mathrm{Rect}(x/2\alpha)$. It is not difficult to show that $\int_{-\infty}^{\infty} F(k)G(k)\mathrm{d}k = 2\pi \int_{-\infty}^{\infty} f(x)g(-x)\mathrm{d}x$. Finally, taking note of the fact that the integrand in Eq.(4) is an even function of k, we write

$$\int_0^\infty \frac{\mathrm{d}}{\mathrm{d}k}[\sin(kR)/k] \times \frac{\mathrm{d}}{\mathrm{d}k}[\sin(kr)/k]\mathrm{d}k = \pi \int_{-\infty}^{\infty}(-\mathrm{i}x/2)\mathrm{Rect}(x/2R) \times (\mathrm{i}x/2)\mathrm{Rect}(-x/2r)\mathrm{d}x$$
$$= (\pi/4) \int_{-\min(r,R)}^{\min(r,R)} x^2 \mathrm{d}x = (\pi/6)\min(r^3, R^3). \tag{5}$$

Substitution into Eq.(4), and also replacing $\hat{\boldsymbol{z}} \times \hat{\boldsymbol{r}}$ with $\sin\theta\,\hat{\boldsymbol{\phi}}$, now yields

$$\boldsymbol{A}(\boldsymbol{r}, t) = \begin{cases} M_0 r \sin\theta\,\hat{\boldsymbol{\phi}}/3\,; & r \le R, \\ M_0 R^3 \sin\theta\,\hat{\boldsymbol{\phi}}/(3r^2)\,; & r \ge R. \end{cases} \tag{6}$$