33) a) $\vec{\nabla}\cdot\vec{A}+\frac{1}{c^2}\frac{\partial\psi}{\partial t}=0 \Rightarrow ik_0\vec{\sigma}\cdot\vec{A}_0-\frac{i\omega}{c^2}\psi_0=0 \Rightarrow \boxed{\vec{\sigma}\cdot\vec{A}_0=\psi_0/c}$ ✓

We have used the relation $k_0=\omega/c$ in arriving at the above formula.

b) $\vec{E}(\vec{r},t)=-\vec{\nabla}\psi-\frac{\partial\vec{A}}{\partial t}=-(ik_0\psi_0\vec{\sigma}-i\omega\vec{A}_0)e^{i(k_0\vec{\sigma}\cdot\vec{r}-\omega t)} \Rightarrow$

$\vec{E}(\vec{r},t)=i\omega(\vec{A}_0-\frac{\psi_0}{c}\vec{\sigma})\exp[i(k_0\vec{\sigma}\cdot\vec{r}-\omega t)]$ ✓

$\vec{H}(\vec{r},t)=\frac{1}{\mu_0}\vec{\nabla}\times\vec{A}=\frac{1}{\mu_0}ik_0\vec{\sigma}\times\vec{A}_0\exp[i(k_0\vec{\sigma}\cdot\vec{r}-\omega t)]$ ✓

c) ① $\vec{\nabla}\cdot\vec{D}=\rho_{free} \Rightarrow \vec{\nabla}\cdot\vec{E}=0 \Rightarrow \vec{\sigma}\cdot\vec{E}=0 \Rightarrow \vec{\sigma}\cdot(\vec{A}_0-\frac{\psi_0}{c}\vec{\sigma})=0 \Rightarrow$

$\vec{\sigma}\cdot\vec{A}_0=\frac{\psi_0}{c}\vec{\sigma}\cdot\vec{\sigma} \Rightarrow \vec{\sigma}\cdot\vec{A}_0=\frac{\psi_0}{c}$ ✓ This is satisfied because of the Lorentz gauge.

② $\vec{\nabla}\times\vec{H}=\vec{J}_{free}+\frac{\partial\vec{D}}{\partial t} \Rightarrow (1/\mu_0)(ik_0)^2\vec{\sigma}\times(\vec{\sigma}\times\vec{A}_0)=-i\omega\epsilon_0[i\omega(\vec{A}_0-\frac{\psi_0}{c}\vec{\sigma})] \Rightarrow$

$-\frac{k_0^2}{\mu_0}[(\vec{\sigma}\cdot\vec{A}_0)\vec{\sigma}-(\vec{\sigma}\cdot\vec{\sigma})\vec{A}_0]=\omega^2\epsilon_0(\vec{A}_0-\frac{\psi_0}{c}\vec{\sigma}) \xrightarrow[k_0=\omega/c]{\mu_0\epsilon_0=1/c^2} (\vec{\sigma}\cdot\vec{\sigma})\vec{A}_0-(\vec{\sigma}\cdot\vec{A}_0)\vec{\sigma}=\vec{A}_0-\frac{\psi_0}{c}\vec{\sigma}$

But $\vec{\sigma}\cdot\vec{\sigma}=1$ and $\vec{\sigma}\cdot\vec{A}_0=\psi_0/c$ (Lorentz gauge). Therefore Maxwell's 2nd is satisfied.

③ $\vec{\nabla}\times\vec{E}=-\frac{\partial\vec{B}}{\partial t} \Rightarrow ik_0\vec{\sigma}\times[i\omega(\vec{A}_0-\frac{\psi_0}{c}\vec{\sigma})]=i\omega\mu_0[\frac{1}{\mu_0}ik_0\vec{\sigma}\times\vec{A}_0] \Rightarrow$

$\cancel{i^2k_0\omega}(\vec{\sigma}\times\vec{A}_0-\frac{\psi_0}{c}\underbrace{\cancel{\vec{\sigma}\times\vec{\sigma}}}_{0})=\cancel{i^2k_0\omega}\,\vec{\sigma}\times\vec{A}_0 \Rightarrow \vec{\sigma}\times\vec{A}_0=\vec{\sigma}\times\vec{A}_0$ ✓

④ $\vec{\nabla}\cdot\vec{B}=0 \Rightarrow ik_0\vec{\sigma}\cdot(ik_0\vec{\sigma}\times\vec{A}_0)=0 \Rightarrow \vec{\sigma}\cdot(\vec{\sigma}\times\vec{A}_0)=(\underbrace{\cancel{\vec{\sigma}\times\vec{\sigma}}}_{0})\cdot\vec{A}_0=0$ ✓