30) a) In the region between the two cylinders, $R_1 < \rho < R_2$, we'll have:

$$\vec{E}(\rho,t)=\tfrac{1}{4}k_0 Z_0 \hat{z}\{I_{10}J_0(k_0R_1)[Y_0(k_0\rho)\cos\omega t - J_0(k_0\rho)\sin\omega t] +$$

$$I_{20}[Y_0(k_0R_2)\cos\omega t - J_0(k_0R_2)\sin\omega t]J_0(k_0\rho)\}$$

Boundary Condition: $\vec{E}(R_1,t)=\vec{E}(R_2,t)=0 \Rightarrow$

$$\begin{array}{l} R_2 \rightarrow \\ R_1 \rightarrow \end{array}\begin{cases} I_{10}J_0(k_0R_1)[Y_0(k_0R_2)\cos\omega t - J_0(k_0R_2)\sin\omega t] + I_{20}[Y_0(k_0R_2)\cos\omega t - J_0(k_0R_2)\sin\omega t]J_0(k_0R_2)=0 \\ I_{10}J_0(k_0R_1)[Y_0(k_0R_1)\cos\omega t - J_0(k_0R_1)\sin\omega t] + I_{20}[Y_0(k_0R_2)\cos\omega t - J_0(k_0R_2)\sin\omega t]J_0(k_0R_1)=0 \end{cases}$$

$$\Rightarrow \begin{cases} [I_{10}J_0(k_0R_1)+I_{20}J_0(k_0R_2)][Y_0(k_0R_2)\cos\omega t - J_0(k_0R_2)\sin\omega t]=0 \\ \{[I_{10}Y_0(k_0R_1)+I_{20}Y_0(k_0R_2)]\cos\omega t - [I_{10}J_0(k_0R_1)+I_{20}J_0(k_0R_2)]\sin\omega t\}J_0(k_0R_1)=0 \end{cases}$$

$$\Rightarrow \begin{cases} I_{10}J_0(k_0R_1)+I_{20}J_0(k_0R_2)=0 \\ I_{10}Y_0(k_0R_1)+I_{20}Y_0(k_0R_2)=0 \end{cases} \Rightarrow \begin{cases} I_{10}/I_{20} = -J_0(k_0R_2)/J_0(k_0R_1) \\ I_{10}/I_{20} = -Y_0(k_0R_2)/Y_0(k_0R_1) \end{cases}$$

A standing wave will thus exist in the cavity between the two cylinders if the following condition is satisfied:

$$\frac{J_0(k_0R_2)}{J_0(k_0R_1)} = \frac{Y_0(k_0R_2)}{Y_0(k_0R_1)} \quad \checkmark$$

As for the magnetic field within the cavity between the two cylinders, we have

$$\vec{H}(\rho,t) = -\frac{1}{4}\, k_0 \hat{\Phi} \Big\{ I_{10} J_0(k_0R_1)\big[Y_1(k_0\rho)\sin\omega t + J_1(k_0\rho)\cos\omega t\big] +$$
$$I_{20}\big[Y_0(k_0R_2)\sin\omega t + J_0(k_0R_2)\cos\omega t\big] J_1(k_0\rho)\Big\}$$

The magnetic field at the cylinder surfaces will be:

$$\vec{H}(R_1,t) = -\frac{1}{4}\, k_0 \hat{\Phi} \Big\{ \big[I_{10}J_0(k_0R_1)Y_1(k_0R_1) + I_{20}J_1(k_0R_1)Y_0(k_0R_2)\big]\sin\omega t +$$
$$\cancelto{0}{\big[I_{10}J_0(k_0R_1) + I_{20}J_0(k_0R_2)\big]} J_1(k_0R_1)\cos\omega t \Big\} \Rightarrow$$

$$\vec{H}(R_1,t) = -\frac{1}{4}\, k_0 I_{10} \hat{\Phi} \big[J_0(k_0R_1)Y_1(k_0R_1) - J_1(k_0R_1)Y_0(k_0R_1)\big]\sin(\omega t)$$
$$= \frac{1}{4}\, k_0 I_{10} \hat{\Phi} \left(\frac{2}{\pi k_0 R_1}\right)\sin(\omega t) = \frac{I_{10}\sin(\omega t)}{2\pi R_1}\hat{\Phi} = J_{s1}(t)\hat{\Phi} \quad \checkmark$$

$$\vec{H}(R_2,t) = -\frac{1}{4}\, k_0 \hat{\Phi} \Big\{ \big[I_{10}J_0(k_0R_1)Y_1(k_0R_2) + I_{20}Y_0(k_0R_2)J_1(k_0R_2)\big]\sin(\omega t) +$$
$$\cancelto{0}{\big[I_{10}J_0(k_0R_1) + I_{20}J_0(k_0R_2)\big]} J_1(k_0R_2)\cos(\omega t) \Big\} \Rightarrow$$

$$\vec{H}(R_2,t) = -\frac{1}{4}\, k_0 \hat{\Phi}\, I_{20} \big[-J_0(k_0R_2)Y_1(k_0R_2) + Y_0(k_0R_2)J_1(k_0R_2)\big]\sin\omega t$$
$$= -\frac{1}{4}\, k_0 I_{20} \hat{\Phi} \left(\frac{2}{\pi k_0 R_2}\right)\sin\omega t = -\frac{I_{20}\sin(\omega t)}{2\pi R_2}\hat{\Phi} = -J_{s2}(t)\hat{\Phi} \quad \checkmark$$

Clearly the H-field on the surface of the conductors is equal to $J_s(t)$ and perpendicular to the direction of the current.

b) Inside the small cylinder $\rho < R_1 < R_2$. The fields are given by:

$$\vec{E}(\rho,t) = \frac{1}{4} k_0 Z_0 \hat{z} \Big\{ I_{10}\big[Y_0(k_0R_1)\cos\omega t - J_0(k_0R_1)\sin\omega t\big] J_0(k_0\rho) +$$

$$I_{20}\big[Y_0(k_0R_2)\cos\omega t - J_0(k_0R_2)\sin\omega t\big] J_0(k_0\rho)\Big\}$$

$$= \frac{1}{4} k_0 Z_0 \hat{z} \Big\{ \cancelto{0}{\big[I_{10} Y_0(k_0R_1) + I_{20} Y_0(k_0R_2)\big]} \cos\omega t - \cancelto{0}{\big[I_{10} J_0(k_0R_1) + I_{20} J_0(k_0R_2)\big]} \sin\omega t \Big\} J_0(k_0\rho)$$

$$= 0 \quad ✓$$

$$\vec{H}(\rho,t) = -\frac{1}{4} k_0 \hat{\phi} J_1(k_0\rho) \Big\{ \cancelto{0}{\big[I_{10} Y_0(k_0R_1) + I_{20} Y_0(k_0R_2)\big]} \sin\omega t + \cancelto{0}{\big[I_{10} J_0(k_0R_1) + I_{20} J_0(k_0R_2)\big]} \cos\omega t \Big\}$$

$$= 0 \quad ✓$$

Outside the large cylinder $\rho > R_2 > R_1$. The fields are given by:

$$\vec{E}(\rho,t) = \frac{1}{4} k_0 Z_0 \hat{z} \cancelto{0}{\big\{ I_{10} J_0(k_0R_1) + I_{20} J_0(k_0R_2) \big\}} \big[Y_0(k_0\rho)\cos\omega t - J_0(k_0\rho)\sin\omega t\big] = 0 \quad ✓$$

$$\vec{H}(\rho,t) = -\frac{1}{4} k_0 \hat{\phi} \big[Y_1(k_0\rho)\sin\omega t + J_1(k_0\rho)\cos\omega t\big] \cancelto{0}{\big[I_{10} J_0(k_0R_1) + I_{20} J_0(k_0R_2)\big]} = 0 \quad ✓$$

Thus the fields vanish from the inside of the small cylinder as well as the outside of the large cylinder. The entire radiation is then confined to the region between the two cylinders. This problem is the analogue of the preceding problem, where the cavity was the region between two plane-parallel mirrors. The resonance condition in the case of plane-parallel mirrors was that the distance between mirrors must be an integer-multiple of $\lambda_0/2$. Here the resonance condition is the relation $J_0(k_0R_2)/J_0(k_0R_1) = Y_0(k_0R_2)/Y_0(k_0R_1)$. Only those values of R_1 and R_2 that satisfy the resonance condition can trap an electromagnetic field between the two cylinders. Note that the ratio of the currents, I_{10}/I_{20}, which is equal to $-J_0(k_0R_2)/J_0(k_0R_1)$ can be either positive or negative, depending on the values of R_1, R_2, λ_0.