Problem 29)

Example 8: Replace $\sin(k_0 x-\omega t)$ with $\mathrm{Re}\{-i e^{ik_0(x-ct)}\}$ to obtain

$$\begin{cases} \vec{E}(\vec{r},t)=\mathrm{Re}\left\{-\frac{i}{2}Z_0 J_{s0}\hat{z}\exp[ik_0(x-ct)]\right\}; & x\geq 0 \\ H(\vec{r},t)=\mathrm{Re}\left\{+\frac{i}{2}J_{s0}\hat{y}\exp[ik_0(x-ct)]\right\}; & x\geq 0 \end{cases}$$

$$\vec{\sigma}=\hat{x}\ ;\ \vec{E}_0=-\frac{i}{2}Z_0 J_{s0}\hat{z};\ \ \vec{H}_0=+\frac{i}{2}J_{s0}\hat{y}$$

$$\langle\vec{S}(\vec{r},t)\rangle=\frac{1}{2}\mathrm{Re}(\vec{E}\times\vec{H}^*)=\frac{1}{2}\mathrm{Re}\left\{-\frac{i}{2}Z_0 J_{s0}\hat{z}e^{ik_0(x-ct)}\times\frac{-i}{2}J_{s0}\hat{y}e^{-ik_0(x-ct)}\right\}\Rightarrow$$

$$\langle S(\vec{r},t)\rangle=\frac{1}{8}Z_0 J_{s0}^2\hat{x}$$ note: $i^2=-1$ and $\hat{z}\times\hat{y}=-\hat{x}$

Problem 19: We will write $\vec{E}$ and $\vec{H}$ as complex entities, dropping the "Re" symbol, unless it is absolutely necessary. Again $\sin(\cdot)$ will be replaced by $\mathrm{Re}(-ie^{i\cdots})$ and $\cos(\cdot)$ will be replaced by $\mathrm{Re}(e^{i\cdots})$.

Case I – $\kappa<k_0$:

$$\begin{cases} \vec{E}(\vec{r},t)=+\frac{i}{2}Z_0 J_{s0}\left(\frac{\kappa}{k_0}\hat{x}-\sqrt{1-(\kappa/k_0)^2}\,\hat{z}\right)e^{ik_0\left[\sqrt{1-(\kappa/k_0)^2}\,x+\frac{\kappa}{k_0}z-ct\right]}; & x\geq 0 \\ H(\vec{r},t)=+\frac{i}{2}J_{s0}\hat{y}e^{ik_0\left[\sqrt{1-(\kappa/k_0)^2}\,x+\frac{\kappa}{k_0}z-ct\right]}; & x\geq 0 \end{cases}$$

$$\vec{\sigma}=\sqrt{1-(\kappa/k_0)^2}\,\hat{x}+(\kappa/k_0)\hat{z};\quad \vec{E}_0=\frac{i}{2}Z_0 J_{s0}\left(\frac{\kappa}{k_0}\hat{x}-\sqrt{1-(\kappa/k_0)^2}\,\hat{z}\right);\ \ \vec{H}_0=\frac{i}{2}J_{s0}\hat{y}.$$

$$\langle\vec{S}(\vec{r},t)\rangle=\frac{1}{2}\mathrm{Re}(\vec{E}\times\vec{H}^*)=\frac{1}{2}\mathrm{Re}(\vec{E}_0\times\vec{H}_0^*)=\frac{1}{8}Z_0 J_{s0}^2\left(\sqrt{1-(\kappa/k_0)^2}\,\hat{x}+\frac{\kappa}{k_0}\hat{z}\right)$$

Case II – $\kappa>k_0$:

$$\begin{cases} \vec{E}(\vec{r},t)=\frac{1}{2}Z_0 J_{s0}\left[\frac{i\kappa}{k_0}\hat{x}+\sqrt{(\kappa/k_0)^2-1}\,\hat{z}\right]e^{+ik_0\left[i\sqrt{(\kappa/k_0)^2-1}\,x+\frac{\kappa}{k_0}z-ct\right]}; & x\geq 0 \\ \vec{H}(\vec{r},t)=\frac{i}{2}J_{s0}\hat{y}e^{ik_0\left[i\sqrt{(\kappa/k_0)^2-1}\,x+(\kappa/k_0)z-ct\right]}; & x\geq 0 \end{cases}$$

$$\vec{\sigma}=i\sqrt{(\kappa/k_0)^2-1}\,\hat{x}+(\kappa/k_0)\hat{z};\ \ \vec{E}_0=\frac{1}{2}Z_0 J_{s0}\left\{i(\kappa/k_0)\hat{x}+\sqrt{(\kappa/k_0)^2-1}\,\hat{z}\right\};\ \ \vec{H}_0=\frac{i}{2}J_{s0}\hat{y}.$$

$$\langle \vec{S}(\vec{r},t)\rangle = \frac{1}{2}\mathrm{Re}(\vec{E}\times\vec{H}^*) = \frac{1}{2}\mathrm{Re}\Big\{\vec{E}_0 e^{ik_0[i\sqrt{(K/k_0)^2-1}\,x+(K/k_0)z-ct]} \times$$

$$\vec{H}_0^* e^{-ik_0[-i\sqrt{(K/k_0)^2-1}\,x+(K/k_0)z-ct]}\Big\} = \frac{1}{2}\mathrm{Re}(\vec{E}_0\times\vec{H}_0^*)e^{-2k_0\sqrt{(K/k_0)^2-1}\,x}$$

$$= \frac{1}{8}Z_0 J_{s_0}^2\,\mathrm{Re}\Big\{\big[i(K/k_0)\hat{x}+\sqrt{(K/k_0)^2-1}\,\hat{z}\big]\times(-i\hat{y})\Big\}e^{-2k_0\sqrt{(K/k_0)^2-1}\,x} \Rightarrow$$

$$\langle \vec{S}(\vec{r},t)\rangle = \frac{K Z_0 J_{s_0}^2}{8k_0}\exp\left(-2k_0\sqrt{(K/k_0)^2-1}\,x\right)\hat{z}; \qquad x \geq 0$$

Problem 20: Case I: $K < k_0$:

$$\vec{E}(\vec{r},t) = \frac{-iZ_0 J_{s_0}\hat{z}}{2\sqrt{1-(K/k_0)^2}}e^{ik_0\left(\sqrt{1-(K/k_0)^2}\,x+\frac{K}{k_0}y-ct\right)}; \qquad x\geq 0$$

$$H(\vec{r},t) = \frac{-iJ_{s_0}}{2\sqrt{1-(K/k_0)^2}}\left(\frac{K}{k_0}\hat{x}-\sqrt{1-(K/k_0)^2}\,\hat{y}\right)e^{ik_0\left(\sqrt{1-(K/k_0)^2}\,x+\frac{K}{k_0}y-ct\right)}; \qquad x\geq 0$$

$$\vec{\sigma} = \sqrt{1-(K/k_0)^2}\,\hat{x}+\frac{K}{k_0}\hat{y}; \quad \vec{E}_0 = -\frac{iZ_0 J_{s_0}}{2\sqrt{1-(K/k_0)^2}}\hat{z}; \quad \vec{H}_0 = -\frac{iJ_{s_0}}{2\sqrt{1-(K/k_0)^2}}\left(\frac{K}{k_0}\hat{x}-\sqrt{1-(K/k_0)^2}\,\hat{y}\right)$$

$$\langle \vec{S}(\vec{r},t)\rangle = \frac{1}{2}\mathrm{Re}(\vec{E}\times\vec{H}^*) = \frac{1}{2}\mathrm{Re}(\vec{E}_0\times\vec{H}_0^*) = \frac{Z_0 J_{s_0}^2}{8[1-(K/k_0)^2]}\mathrm{Re}\left\{i\hat{z}\times(-i)\left[\frac{K}{k_0}\hat{x}-\sqrt{1-(K/k_0)^2}\,\hat{y}\right]\right\}$$

$$\Rightarrow \langle \vec{S}(\vec{r},t)\rangle = \frac{Z_0 J_{s_0}^2}{8[1-(K/k_0)^2]}\left(\sqrt{1-(K/k_0)^2}\,\hat{x}+\frac{K}{k_0}\hat{y}\right) \qquad x\geq 0$$

Case II - $K > k_0$)

$$\vec{E}(\vec{r},t) = -\frac{Z_0 J_{s_0}\hat{z}}{2\sqrt{(K/k_0)^2-1}}e^{ik_0\left(i\sqrt{(K/k_0)^2-1}\,x+\frac{K}{k_0}y-ct\right)}; \qquad x\geq 0$$

$$H(\vec{r},t) = -\frac{J_{s_0}}{2\sqrt{(K/k_0)^2-1}}\left\{(K/k_0)\hat{x}-i\sqrt{(K/k_0)^2-1}\,\hat{y}\right\}e^{ik_0\left(i\sqrt{(K/k_0)^2-1}\,x+\frac{K}{k_0}y-ct\right)}; \qquad x\geq 0$$

$$\vec{\sigma} = i\sqrt{(K/k_0)^2-1}\,\hat{x}+(K/k_0)\hat{y}; \quad \vec{E}_0 = -\frac{Z_0 J_{s_0}}{2\sqrt{(K/k_0)^2-1}}\hat{z}; \quad \vec{H}_0 = -\frac{J_{s_0}}{2\sqrt{(K/k_0)^2-1}}\left[\frac{K}{k_0}\hat{x}-i\sqrt{(K/k_0)^2-1}\,\hat{y}\right].$$

$$\langle \vec{S}(\vec{r},t)\rangle = \frac{1}{2}\mathrm{Re}(\vec{E}\times\vec{H}^{*}) = \frac{1}{2}\mathrm{Re}(\vec{E}_0\times\vec{H}_0^{*})\,e^{-2k_0\sqrt{(\kappa/k_0)^2-1}\,x} =$$

$$\frac{Z_0 J_{s0}^2}{8[(\kappa/k_0)^2-1]}\,e^{-2k_0\sqrt{(\kappa/k_0)^2-1}\,x}\,\mathrm{Re}\left\{\hat{z}\times\left[\frac{\kappa}{k_0}\hat{x}+i\sqrt{(\kappa/k_0)^2-1}\,\hat{y}\right]\right\} \Rightarrow$$

$$\langle \vec{S}(\vec{r},t)\rangle = \frac{(\kappa/k_0)Z_0 J_{s0}^2}{8[(\kappa/k_0)^2-1]}\exp\left(-2k_0\sqrt{(\kappa/k_0)^2-1}\,x\right)\hat{y}; \quad x\geq 0$$