Problem 28)

a) The large-argument approximate forms of Bessel functions of the first and second kinds are

$$J_n(x) \approx \sqrt{2/(\pi x)}\,\cos[x-(n\pi/2)-(\pi/4)],$$

$$Y_n(x) \approx \sqrt{2/(\pi x)}\,\sin[x-(n\pi/2)-(\pi/4)].$$

Substitution into the expressions for the *E*- and *H*-fields then yields

$$\boldsymbol{E}(\boldsymbol{r},t) \simeq -\tfrac{1}{4}\mu_o I_o \omega_o \sqrt{2c/(\pi\rho\omega_o)}\,[\cos(\rho\omega_o/c-\pi/4)\cos(\omega_o t)+\sin(\rho\omega_o/c-\pi/4)\sin(\omega_o t)]\hat{\boldsymbol{z}}$$

$$= -\frac{Z_o I_o}{\sqrt{4\lambda_o\rho}}\cos[\omega_o(t-\rho/c)+\pi/4]\hat{\boldsymbol{z}}.$$

$$\boldsymbol{H}(\boldsymbol{r},t) \simeq \frac{I_o\omega_o}{4c}\sqrt{2c/(\pi\rho\omega_o)}\,[\cos(\rho\omega_o/c-3\pi/4)\sin(\omega_o t)-\sin(\rho\omega_o/c-3\pi/4)\cos(\omega_o t)]\hat{\boldsymbol{\phi}}$$

$$= \frac{I_o}{\sqrt{4\lambda_o\rho}}\cos[\omega_o(t-\rho/c)+\pi/4]\hat{\boldsymbol{\phi}}.$$

b) $$\boldsymbol{S}(\boldsymbol{r},t)=\boldsymbol{E}(\boldsymbol{r},t)\times\boldsymbol{H}(\boldsymbol{r},t)\simeq\frac{Z_o I_o^2}{4\lambda_o\rho}\cos^2[\omega_o(t-\rho/c)+\pi/4]\hat{\boldsymbol{\rho}} \qquad \text{(far field).}$$

c) $$\langle\boldsymbol{S}(\boldsymbol{r},t)\rangle\simeq\frac{Z_o I_o^2}{4\lambda_o\rho}\langle\cos^2[\omega_o(t-\rho/c)+\pi/4]\rangle\hat{\boldsymbol{\rho}}=\frac{Z_o I_o^2}{8\lambda_o\rho}\hat{\boldsymbol{\rho}} \qquad \text{(far field).}$$

The time-averaged energy leaving a cylinder of radius R and height L per second is obtained by multiplying the above time-averaged Poynting vector at $\rho=R$ with the surface area $2\pi RL$ of the cylinder. The result, $\pi Z_o I_o^2 L/(4\lambda_o)$, is clearly independent of the cylinder radius, as it should be, considering that the electromagnetic power radiated by the wire must leave the surrounding cylinder, irrespective of the cylinder radius.