Opti 501 **Solutions**

Problem 27)

a) $\boldsymbol{P}(\boldsymbol{r},t) = p_o\delta(x)\,\delta(y)\,\delta(z)[\cos(\omega_o t)\hat{\boldsymbol{x}} + \sin(\omega_o t)\hat{\boldsymbol{y}}]$.

b) $\rho_b^{(e)}(\boldsymbol{r},t) = -\boldsymbol{\nabla}\cdot\boldsymbol{P}(\boldsymbol{r},t) = -\partial P_x/\partial x - \partial P_y/\partial y = -p_o[\delta'(x)\,\delta(y)\cos(\omega_o t) + \delta(x)\,\delta'(y)\sin(\omega_o t)]\,\delta(z)$.

$\boldsymbol{J}_b^{(e)}(\boldsymbol{r},t) = \partial\boldsymbol{P}(\boldsymbol{r},t)/\partial t = -p_o\omega_o\,\delta(x)\,\delta(y)\,\delta(z)[\sin(\omega_o t)\hat{\boldsymbol{x}} - \cos(\omega_o t)\hat{\boldsymbol{y}}]$.

c) Continuity equation:

$$\boldsymbol{\nabla}\cdot\boldsymbol{J}_b^{(e)} + \partial\rho_b^{(e)}/\partial t = -p_o\omega_o[\delta'(x)\,\delta(y)\,\delta(z)\sin(\omega_o t) - \delta(x)\,\delta'(y)\,\delta(z)\cos(\omega_o t)]$$
$$-p_o\omega_o[-\delta'(x)\,\delta(y)\sin(\omega_o t) + \delta(x)\,\delta'(y)\cos(\omega_o t)]\,\delta(z) = 0.$$

d) For the electric point-dipole $p_o\cos(\omega_o t)\hat{\boldsymbol{z}}$ aligned with the z-axis, the potentials are given in a spherical coordinate system. In the present problem, however, we have two oscillating dipoles, one aligned with the x-axis, having magnitude $p_o\cos(\omega_o t)$, the other aligned with the y-axis and having magnitude $p_o\sin(\omega_o t)$. Retaining the same spherical coordinate system in which θ is measured from the z-axis, we recognize that, for the first dipole, $\cos\theta$ in the expression of the scalar potential must be replaced with $\sin\theta\cos\phi$, while for the second dipole it must be replaced with $\sin\theta\sin\phi$. Also, for the second dipole, the origin of time t must be shifted by one quarter of one period such that $\cos(\omega_o t)$ is turned into $\sin(\omega_o t)$, in which case $\sin(\omega_o t)$ appearing in the expressions of $\psi(\boldsymbol{r},t)$ and $\boldsymbol{A}(\boldsymbol{r},t)$ must undergo a corresponding shift to become $-\cos(\omega_o t)$. Subsequently we add the respective potentials of the two dipoles to obtain

$$\boldsymbol{A}(\boldsymbol{r},t) = -(\mu_o p_o\omega_o/4\pi r)\{\sin[\omega_o(t-r/c)]\hat{\boldsymbol{x}} - \cos[\omega_o(t-r/c)]\hat{\boldsymbol{y}}\};$$

$$\psi(\boldsymbol{r},t) = (p_o\sin\theta\cos\phi/4\pi\varepsilon_o r^2)\{\cos[\omega_o(t-r/c)] - (\omega_o r/c)\sin[\omega_o(t-r/c)]\}$$
$$+ (p_o\sin\theta\sin\phi/4\pi\varepsilon_o r^2)\{\sin[\omega_o(t-r/c)] + (\omega_o r/c)\cos[\omega_o(t-r/c)]\}$$
$$= (p_o\sin\theta/4\pi\varepsilon_o r^2)\{\cos[\omega_o(t-r/c)-\phi] - (\omega_o r/c)\sin[\omega_o(t-r/c)-\phi]\}.$$