

Problem 18)

a) $\mathbf{M}(\mathbf{r}, t) = m_0 \delta(x) \delta(y) \delta(z) [\cos(\omega_0 t) \hat{\mathbf{x}} + \sin(\omega_0 t) \hat{\mathbf{y}}]$.

b) $\mathbf{J}_b^{(e)}(\mathbf{r}, t) = \mu_0^{-1} \nabla \times \mathbf{M}(\mathbf{r}, t) = \mu_0^{-1} [-(\partial M_y / \partial z) \hat{\mathbf{x}} + (\partial M_x / \partial z) \hat{\mathbf{y}} + (\partial M_y / \partial x - \partial M_x / \partial y) \hat{\mathbf{z}}]$
 $= \mu_0^{-1} m_0 \{ \delta(x) \delta(y) \delta'(z) [-\sin(\omega_0 t) \hat{\mathbf{x}} + \cos(\omega_0 t) \hat{\mathbf{y}}]$
 $+ [\delta'(x) \delta(y) \sin(\omega_0 t) - \delta(x) \delta'(y) \cos(\omega_0 t)] \delta(z) \hat{\mathbf{z}} \}.$

c) For the magnetic point-dipole $m_0 \cos(\omega_0 t) \hat{\mathbf{z}}$ aligned with the z -axis, the vector potential is given in a spherical coordinate system. In the present problem we have two oscillating dipoles, one aligned with the x -axis, having magnitude $m_0 \cos(\omega_0 t)$, the other aligned with the y -axis and having magnitude $m_0 \sin(\omega_0 t)$. Retaining the same spherical coordinate system in which θ is measured from the z -axis, we recognize that, for the first dipole, $(\sin \theta) \hat{\boldsymbol{\phi}} = \hat{\mathbf{z}} \times \hat{\mathbf{r}}$ in the expression of the vector potential of a z -oriented dipole must be replaced with $\hat{\mathbf{x}} \times \hat{\mathbf{r}}$, while for the second dipole it must be replaced with $\hat{\mathbf{y}} \times \hat{\mathbf{r}}$. Also, for the second dipole, the origin of time t must be shifted by one quarter of one period such that $\cos(\omega_0 t)$ is turned into $\sin(\omega_0 t)$, in which case $\sin(\omega_0 t)$ appearing in the expression of the vector potential must undergo a corresponding shift to become $-\cos(\omega_0 t)$. Adding the vector potentials of the two dipoles we find

$$\begin{aligned} \mathbf{A}(\mathbf{r}, t) &= (m_0 / 4\pi r^2) \{ \cos[\omega_0(t - r/c)] - (\omega_0 r/c) \sin[\omega_0(t - r/c)] \} (\hat{\mathbf{x}} \times \hat{\mathbf{r}}) \\ &\quad + (m_0 / 4\pi r^2) \{ \sin[\omega_0(t - r/c)] + (\omega_0 r/c) \cos[\omega_0(t - r/c)] \} (\hat{\mathbf{y}} \times \hat{\mathbf{r}}) \\ &= (m_0 \hat{\mathbf{r}} / 4\pi r^2) \times \{ [(\omega_0 r/c) \hat{\mathbf{x}} - \hat{\mathbf{y}}] \sin[\omega_0(t - r/c)] - [\hat{\mathbf{x}} + (\omega_0 r/c) \hat{\mathbf{y}}] \cos[\omega_0(t - r/c)] \}. \end{aligned}$$