Problem 9)

$$\vec J(\vec r) = J_{s_0}\,\delta(\rho-R)\,\hat\phi \Rightarrow \vec J(\vec k) = \int_{-\infty}^{\infty} J_{s_0}\,\delta(\rho-R)\,\underbrace{(\hat z\times\hat\rho)}_{\hat\phi}\, e^{-i\vec k\cdot\vec r}\, d\vec r$$

$$= J_{s_0}\,\hat z\times\int_{z=-\infty}^{\infty}\int_{\rho=0}^{\infty}\int_{\theta=0}^{2\pi}\hat\rho\,\delta(\rho-R)\,e^{-ik_z z}\,e^{-ik_\parallel\rho\cos\theta}\,\rho\, d\theta\, d\rho\, dz$$

$\hat k_\parallel \cos\theta$ $\qquad$ $\vec k_\parallel = k_x\hat x + k_y\hat y$

$$= J_{s_0}(\hat z\times\hat k_\parallel)\int_{\rho=0}^{\infty}\rho\,\delta(\rho-R)\Big[\underbrace{\int_{\theta=0}^{2\pi}\cos\theta\, e^{-ik_\parallel\rho\cos\theta}\,d\theta}_{-i2\pi J_1(k_\parallel\rho)}\Big]d\rho\,\underbrace{\int_{-\infty}^{\infty} e^{-ik_z z}\,dz}_{2\pi\,\delta(k_z)}$$

$$= -i(2\pi)^2 J_{s_0}(\hat z\times\hat k_\parallel)\,\delta(k_z)\int_0^{\infty}\rho\,\delta(\rho-R)\,J_1(k_\parallel\rho)\,d\rho = -i(2\pi)^2 J_{s_0} R\,\delta(k_z)\,J_1(k_\parallel R)\,(\hat z\times\hat k_\parallel)$$

$$\vec A(\vec k) = \frac{\mu_0\vec J(\vec k)}{k^2} = -\frac{i(2\pi)^2\mu_0 J_{s_0} R\,\delta(k_z)\,J_1(k_\parallel R)\,(\hat z\times\hat k_\parallel)}{k_\parallel^2+k_z^2}$$

$$\vec A(\vec r) = \frac{1}{(2\pi)^3}\int_{-\infty}^{\infty}\vec A(\vec k)\,e^{+i\vec k\cdot\vec r}\,d\vec k = -\frac{i}{2\pi}\mu_0 J_{s_0}R\,\hat z\times\int_{-\infty}^{\infty}\frac{\hat k_\parallel}{k_\parallel^2}\,J_1(k_\parallel R)\,e^{i\vec k_\parallel\cdot\vec\rho}\,d\vec k_\parallel$$

$$= -\frac{i}{2\pi}\mu_0 J_{s_0}R\,\hat z\times\int_{k_\parallel=0}^{\infty}\int_{\theta=0}^{2\pi}\frac{\hat\rho\cos\theta}{k_\parallel^2}\,J_1(k_\parallel R)\,e^{ik_\parallel\rho\cos\theta}\,k_\parallel\,d\theta\,dk_\parallel$$

$$= -\frac{i}{2\pi}\mu_0 J_{s_0}R\,(\hat z\times\hat\rho)\int_{k_\parallel=0}^{\infty}\frac{J_1(k_\parallel R)}{k_\parallel}\Big(\underbrace{\int_0^{2\pi}\cos\theta\, e^{ik_\parallel\rho\cos\theta}\,d\theta}_{=2\pi i J_1(k_\parallel\rho)}\Big)dk_\parallel$$

$$= \mu_0 J_{s_0}R\,\hat\phi\int_0^{\infty}\frac{J_1(k_\parallel R)\,J_1(k_\parallel\rho)}{k_\parallel}\,dk_\parallel = \mu_0 J_{s_0}R\,\hat\phi\begin{cases}\rho/(2R); & \rho<R\\ R/(2\rho); & \rho>R\end{cases}$$

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$$\Rightarrow \vec A(\vec r) = \begin{cases}\tfrac{1}{2}\mu_0 J_{s_0}\rho\,\hat\phi; & \rho<R \text{ (inside)}\\ \tfrac{1}{2}\mu_0 J_{s_0}R^2\rho^{-1}\hat\phi; & \rho>R \text{ (outside)}\end{cases}$$

✓

$$\vec H(\vec r) = \frac{1}{\mu_0}\vec\nabla\times\vec A(\vec r) = \frac{1}{\mu_0\rho}\frac{\partial}{\partial\rho}(\rho A_\phi)\,\hat z = \frac{1}{2\rho}J_{s_0}\hat z\begin{cases}\frac{\partial}{\partial\rho}(\rho^2)\\ R^2\frac{\partial}{\partial\rho}(\rho\rho^{-1})\end{cases} = \begin{cases}J_{s_0}\hat z; & \rho<R\\ 0 & \rho>R\end{cases}$$