Problem 2)

Maxwell's 1st equation: $\vec\nabla\cdot\vec D=\rho_{free} \Rightarrow \epsilon_0\vec\nabla\cdot\vec E=\rho_{free}-\vec\nabla\cdot\vec P \Rightarrow i\epsilon_0\vec k\cdot\vec E(\vec k,\omega)=\rho_{free}(\vec k,\omega) - i\vec k\cdot\vec P(\vec k,\omega) \Rightarrow E_\parallel(\vec k,\omega) = -\frac{i}{\epsilon_0 k}\rho_{free}(\vec k,\omega) - \frac{1}{\epsilon_0}P_\parallel(\vec k,\omega).$

Maxwell's 4th equation: $\vec\nabla\cdot\vec B=0 \Rightarrow \mu_0\vec\nabla\cdot\vec H = -\vec\nabla\cdot\vec M \Rightarrow i\mu_0\vec k\cdot\vec H = -i\vec k\cdot\vec M \Rightarrow$

$$H_\parallel(\vec k,\omega) = -\frac{1}{\mu_0}M_\parallel(\vec k,\omega).$$

Maxwell's 2nd equation: $\vec\nabla\times\vec H = \vec J_{free}+\frac{\partial\vec D}{\partial t} \Rightarrow i\vec k\times\vec H = \vec J_{free} - i\omega(\epsilon_0\vec E+\vec P)\Rightarrow$

Cross-multiply both sides by $\vec k \Rightarrow i\vec k\times(\vec k\times\vec H) = \vec k\times\vec J_{free} - i\omega\vec k\times\vec P - i\epsilon_0\omega\vec k\times\vec E$

$\Rightarrow k^2 H_\perp(\vec k,\omega) = i\vec k\times\vec J_{free}(\vec k,\omega)+\omega\vec k\times\vec P(\vec k,\omega)+\epsilon_0\omega\vec k\times\vec E(\vec k,\omega).$

Now, from Maxwell's 3rd equation we have:

Tangential components of $\vec M$ and $\vec H$ cancel out

$\vec\nabla\times\vec E = -\frac{\partial B}{\partial t} \Rightarrow i\vec k\times\vec E(\vec k,\omega) = i\omega\mu_0\vec H(\vec k,\omega) + i\omega\vec M(\vec k,\omega) \Rightarrow \vec k\times\vec E = \mu_0\omega H_\perp + \omega M_\perp$

Substituting this result in the previous equation yields:

$$k^2\vec H_\perp = i\vec k\times\vec J_{free} + \omega\vec k\times\vec P + \mu_0\epsilon_0\omega^2\vec H_\perp + \epsilon_0\omega^2\vec M_\perp \Rightarrow \vec H_\perp(\vec k,\omega) = \frac{i\vec k\times\vec J_{free}+\omega\vec k\times\vec P+\epsilon_0\omega^2\vec M_\perp}{k^2-(\omega/c)^2}.$$

Finally, cross-multiplying Maxwell's 3rd equation with $\vec k$ yields:

$$\vec k\times(\vec k\times\vec E) = \mu_0\omega\vec k\times\vec H_\perp + \omega\vec k\times\vec M_\perp \Rightarrow -k^2\vec E_\perp(\vec k,\omega) = \frac{-i\mu_0\omega k^2\vec J_\perp - \mu_0\omega^2k^2\vec P_\perp + \mu_0\epsilon_0\omega^3\vec k\times\vec M_\perp}{k^2-(\omega/c)^2} + \omega\vec k\times\vec M_\perp$$

$$\Rightarrow \vec E_\perp(\vec k,\omega) = \frac{i\mu_0\omega\vec J_\perp+\mu_0\omega^2\vec P_\perp - \omega\vec k\times\vec M}{k^2-(\omega/c)^2}.$$