Problem 3.34) a) For an electromagnetic field to be trapped inside a perfectly electrically conducting cavity, the tangential component of the E-field must vanish on all the internal surfaces. Therefore,

$$E_z(r_\parallel = R, \phi, z, t) = E_0\, J_0(\omega_0 R/c)\sin(\omega_0 t) = 0 \quad \rightarrow \quad J_0(\omega_0 R/c) = 0. \tag{1}$$

In words, the cylinder radius must be chosen such that $\omega_0 R/c = 2\pi R/\lambda_0$ is a zero of the Bessel function $J_0(\cdot)$.

b) A surface charge-density exists on the top and bottom facets of the cylindrical can. Invoking Maxwell's first boundary condition, the surface charge-density must equal the perpendicular component of the D-field, namely,

Top and bottom caps: $$\sigma_s(r_\parallel, \phi, z = \pm L/2\,, t) = \mp\varepsilon_0 E_0\, J_0(\omega_0 r_\parallel/c)\sin(\omega_0 t). \tag{2}$$

c) A surface current-density $\boldsymbol{J}_s$ exists wherever the H-field happens to have a component parallel to the surface, that is,

Cylindrical wall: $$\boldsymbol{J}_s(r_\parallel = R, \phi, z, t) = -(E_0/Z_0)\, J_1(\omega_0 R/c)\cos(\omega_0 t)\,\hat{\boldsymbol{z}}. \tag{3}$$

Top and bottom caps: $$\boldsymbol{J}_s(r_\parallel, \phi, z = \pm L/2\,, t) = \pm(E_0/Z_0)\, J_1(\omega_0 r_\parallel/c)\cos(\omega_0 t)\,\hat{\boldsymbol{r}}_\parallel. \tag{4}$$

Note that at $r_\parallel = R$, the current exits the upper cap and enters the cylindrical wall. The opposite happens at the bottom cap, where the current arrives from the cylindrical wall, then enters the cap from the rim located at $r_\parallel = R$.

d) On the cylindrical facet, the charge-density is zero, and so is the divergence of the surface current-density $\boldsymbol{J}_s$ given by Eq.(3). This confirms that, on the interior cylindrical wall, the continuity equation is satisfied.

At the top and bottom caps we have

$$\begin{aligned}
\boldsymbol{\nabla}\cdot\boldsymbol{J}_s(r_\parallel, \phi, z = \pm L/2\,, t) &= \pm(E_0/Z_0)\frac{\partial[r_\parallel J_1(\omega_0 r_\parallel/c)]}{r_\parallel \partial r_\parallel}\cos(\omega_0 t)\\
&= \pm(E_0/Z_0)\frac{J_1(\omega_0 r_\parallel/c) + (\omega_0 r_\parallel/c)J_1'(\omega_0 r_\parallel/c)}{r_\parallel}\cos(\omega_0 t)\\
&= \pm(E_0/Z_0)\frac{(\omega_0 r_\parallel/c)J_0(\omega_0 r_\parallel/c)}{r_\parallel}\cos(\omega_0 t)\\
&= \pm\varepsilon_0 E_0 \omega_0\, J_0(\omega_0 r_\parallel/c)\cos(\omega_0 t).
\end{aligned} \tag{5}$$

Clearly, $\boldsymbol{\nabla}\cdot\boldsymbol{J}_s + \partial\sigma_s/\partial t = 0$; see Eqs.(2) and (5).

Digression: Below we confirm that the $\boldsymbol{E}$ and $\boldsymbol{H}$ fields given in the statement of the problem do in fact satisfy Maxwell's equations.

i) $\boldsymbol{\nabla}\cdot\boldsymbol{D} = \varepsilon_0\,\partial E_z/\partial z = 0$

ii) $\boldsymbol{\nabla}\times\boldsymbol{H}=\frac{\partial(r_\parallel H_\phi)}{r_\parallel\partial r_\parallel}\hat{\boldsymbol{z}}=(E_0/Z_0)\,\frac{\partial[r_\parallel J_1(\omega_0 r_\parallel/c)]}{r_\parallel\partial r_\parallel}\cos(\omega_0 t)\,\hat{\boldsymbol{z}}$

$$=(E_0/Z_0)\frac{J_1(\omega_0 r_\parallel/c)+(\omega_0 r_\parallel/c)J_1'(\omega_0 r_\parallel/c)}{r_\parallel}\cos(\omega_0 t)\,\hat{\boldsymbol{z}}$$

$$=(E_0/Z_0)\frac{(\omega_0 r_\parallel/c)J_0(\omega_0 r_\parallel/c)}{r_\parallel}\cos(\omega_0 t)\,\hat{\boldsymbol{z}}$$

$$=\varepsilon_0 E_0\omega_0\,J_0(\omega_0 r_\parallel/c)\cos(\omega_0 t)\,\hat{\boldsymbol{z}}=\varepsilon_0\,\partial\boldsymbol{E}/\partial t$$

iii) $\boldsymbol{\nabla}\times\boldsymbol{E}=-(\partial E_z/\partial r_\parallel)\hat{\boldsymbol{\phi}}=-E_0(\omega_0/c)\,J_0'(\omega_0 r_\parallel/c)\sin(\omega_0 t)\,\hat{\boldsymbol{\phi}}$ ← $J_0'(x)=-J_1(x)$ Chap.3, Eq.(39)

$$=E_0(\omega_0/c)\,J_1(\omega_0 r_\parallel/c)\sin(\omega_0 t)\,\hat{\boldsymbol{\phi}}=-\mu_0\,\partial\boldsymbol{H}/\partial t.$$

iv) $\boldsymbol{\nabla}\cdot\boldsymbol{B}=\mu_0 r_\parallel^{-1}\,\partial H_\phi/\partial\phi=0.$
