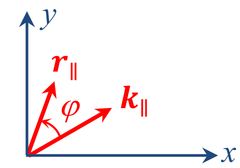
Problem 3.28) The Fourier transform of the charge distribution of the ring is straightforwardly evaluated, as follows:

$$\rho_{\text{free}}(\boldsymbol{k},\omega) = \int_{-\infty}^{\infty} \rho_{\text{free}}(r_\parallel,\phi,z,t)\exp[-i(\boldsymbol{k}\cdot\boldsymbol{r}-\omega t)]\,d\boldsymbol{r}dt$$

$$= \int_{-\infty}^{\infty} \sigma_0\left[\text{Circ}\left(\frac{r_\parallel}{R_2}\right)-\text{Circ}\left(\frac{r_\parallel}{R_1}\right)\right]\delta(z)\exp(-i\boldsymbol{k}_\parallel\cdot\boldsymbol{r}_\parallel)\exp(-ik_z z)\exp(i\omega t)\,d\boldsymbol{r}dt$$

$$= 2\pi\sigma_0\delta(\omega)\int_{R_1}^{R_2}\int_0^{2\pi}\exp(-ik_\parallel r_\parallel\cos\varphi)\,r_\parallel dr_\parallel d\varphi$$



$$= (2\pi)^2\sigma_0\,\delta(\omega)\int_{R_1}^{R_2} r_\parallel J_0(k_\parallel r_\parallel)\,dr_\parallel$$

$$= (2\pi)^2\sigma_0\,\delta(\omega)k_\parallel^{-2}\int_{k_\parallel R_1}^{k_\parallel R_2} x J_0(x)\,dx$$

$$= (2\pi)^2\sigma_0\,\delta(\omega)k_\parallel^{-1}[R_2 J_1(k_\parallel R_2) - R_1 J_1(k_\parallel R_1)].$$