Solution to Problem 26) In the limit of large x, we use Eqs.(37) and (38) to arrive at

$$\begin{aligned} Y_\nu(x)J_{\nu+1}(x) - J_\nu(x)Y_{\nu+1}(x) &\cong \tfrac{2}{\pi x}\sin(x - \tfrac{1}{2}\nu\pi - \tfrac{1}{4}\pi)\cos(x - \tfrac{1}{2}\nu\pi - \tfrac{3}{4}\pi) \\ &\quad - \tfrac{2}{\pi x}\cos(x - \tfrac{1}{2}\nu\pi - \tfrac{1}{4}\pi)\sin(x - \tfrac{1}{2}\nu\pi - \tfrac{3}{4}\pi) \\ &= \tfrac{2}{\pi x}[\sin^2(x - \tfrac{1}{2}\nu\pi - \tfrac{1}{4}\pi) + \cos^2(x - \tfrac{1}{2}\nu\pi - \tfrac{1}{4}\pi)] \\ &= \tfrac{2}{\pi x}. \end{aligned} \qquad (1)$$

Next, we differentiate the function $Y_\nu(x)J_{\nu+1}(x) - J_\nu(x)Y_{\nu+1}(x)$ with respect to x, and use the identities in Eqs.(41) and (42) to write

$$\begin{aligned} &\tfrac{d}{dx}[Y_\nu(x)J_{\nu+1}(x) - J_\nu(x)Y_{\nu+1}(x)] \\ &= Y_\nu'(x)J_{\nu+1}(x) + Y_\nu(x)J_{\nu+1}'(x) - J_\nu'(x)Y_{\nu+1}(x) - J_\nu(x)Y_{\nu+1}'(x) \\ &= \left[\left(\tfrac{\nu}{x}\right)Y_\nu(x) - Y_{\nu+1}(x)\right]J_{\nu+1}(x) + Y_\nu(x)\left[J_\nu(x) - \left(\tfrac{\nu+1}{x}\right)J_{\nu+1}(x)\right] \\ &\quad - \left[\left(\tfrac{\nu}{x}\right)J_\nu(x) - J_{\nu+1}(x)\right]Y_{\nu+1}(x) - J_\nu(x)\left[Y_\nu(x) - \left(\tfrac{\nu+1}{x}\right)Y_{\nu+1}(x)\right] \\ &= -\tfrac{1}{x}[Y_\nu(x)J_{\nu+1}(x) - J_\nu(x)Y_{\nu+1}(x)]. \end{aligned} \qquad (2)$$

The above equation may now be rewritten as follows:

$$[Y_\nu(x)J_{\nu+1}(x) - J_\nu(x)Y_{\nu+1}(x)]'/[Y_\nu(x)J_{\nu+1}(x) - J_\nu(x)Y_{\nu+1}(x)] = -1/x. \qquad (3)$$

Integrating both sides of Eq.(3) with respect to x, and denoting the constant of integration by c, we find

$$\ln[Y_\nu(x)J_{\nu+1}(x) - J_\nu(x)Y_{\nu+1}(x)] = -\ln x + c. \qquad (4)$$

Consequently,

$$Y_\nu(x)J_{\nu+1}(x) - J_\nu(x)Y_{\nu+1}(x) = c'/x, \qquad (5)$$

where $c' = \exp(c)$ signifies another way of writing the constant of integration. A comparison of Eq.(5) with Eq.(1) now reveals that $c' = 2/\pi$, thus completing the proof.