Problem 3-23) The desired integral may be written as follows:

$$\int_0^\infty r J_n(kr)\, J_n(k'r)\mathrm{d}r = \lim_{\alpha\to\infty}\int_0^\infty \frac{\alpha^2}{\alpha^2+r^2} r J_n(kr)\, J_n(k'r)\mathrm{d}r = \alpha^2 I_n(\alpha k) K_n(\alpha k'), \qquad k' > k > 0.$$

We now invoke the large-argument asymptotic forms of $I_n(\cdot)$ and $K_n(\cdot)$ given in G&R 8.451-5,6:

$$I_n(x) \sim \frac{\exp(x)}{\sqrt{2\pi x}}\left[1 - \frac{(2n+1)(2n-1)}{8x} + \frac{(2n+3)(2n+1)(2n-1)(2n-3)}{128x^2} + \cdots\right],$$

$$K_n(x) \sim \frac{\sqrt{\pi}\,\exp(-x)}{\sqrt{2x}}\left[1 + \frac{(2n+1)(2n-1)}{8x} + \frac{(2n+3)(2n+1)(2n-1)(2n-3)}{128x^2} + \cdots\right].$$

For $k \le k'$, we will have

$$\int_0^\infty r J_n(kr)\, J_n(k'r)\mathrm{d}r = \lim_{\alpha\to\infty} \alpha^2 \frac{\exp(\alpha k)}{\sqrt{2\pi\alpha k}}\left[1 - \frac{(2n+1)(2n-1)}{8\alpha k} + \cdots\right] \frac{\sqrt{\pi}\,\exp(-\alpha k')}{\sqrt{2\alpha k'}}\left[1 + \frac{(2n+1)(2n-1)}{8\alpha k'} + \cdots\right]$$

$$= \lim_{\alpha\to\infty} \alpha \frac{\exp[-\alpha(k'-k)]}{2\sqrt{kk'}}\left[1 - \frac{(4n^2-1)(k'-k)}{8\alpha kk'} + \cdots\right].$$

When $k > k'$, the roles of k and k' in the above equation will be reversed, so that, in the final expression, $(k'-k)$ is replaced with $(k-k')$. The resulting $\tfrac{1}{2}\alpha\exp(-\alpha|k-k'|)$ thus approaches a delta-function in the limit $\alpha\to\infty$, while all the high-order terms go to zero. We thus find

$$\int_0^\infty r J_n(kr)\, J_n(k'r)\mathrm{d}r = (kk')^{-1/2}\,\delta(k-k').$$